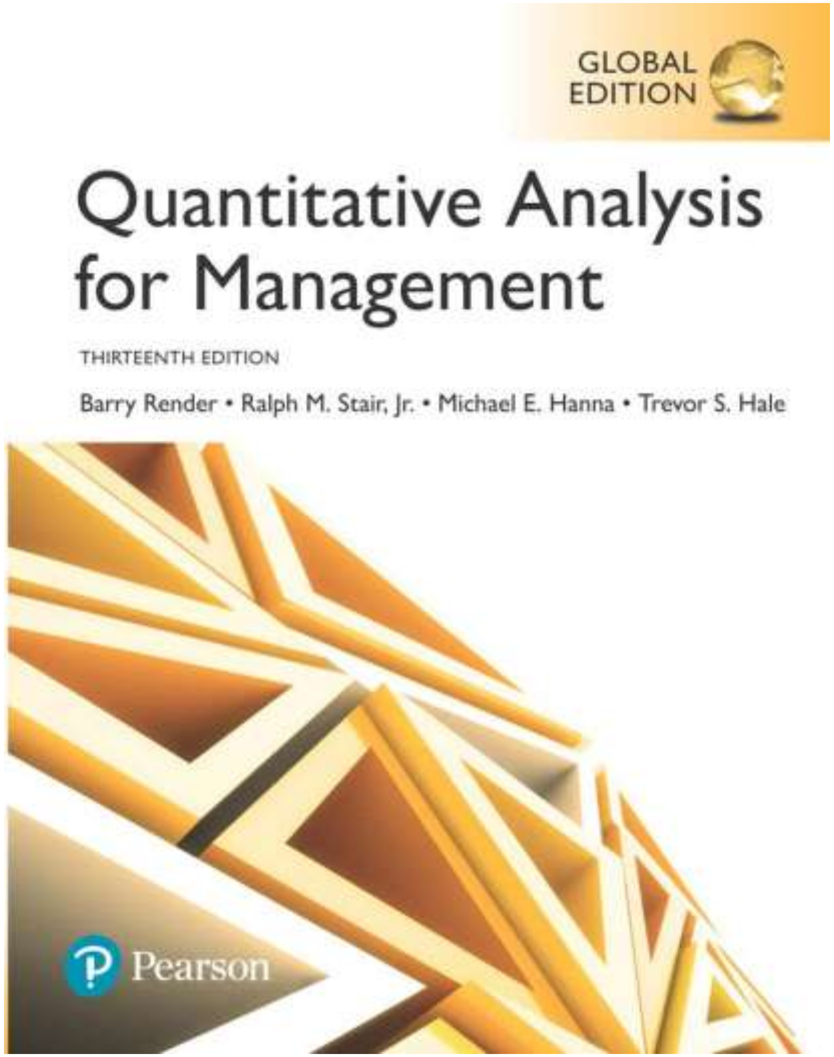
Quantitative Analysis for Management

Thirteenth Edition, Global Edition



Chapter 1

Introduction to Quantitative Analysis

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Introduction

- Mathematical tools have been used for thousands of years
- Quantitative analysis can be applied to a wide variety of problems
  - Not enough to just know the mathematics of a technique
  - Must understand the specific applicability of the technique, its limitations, and assumptions
  - Successful use of quantitative techniques usually results in a solution that is timely, accurate, flexible, economical, reliable, and easy to understand and use

Examples of Quantitative Analyses

- Taco Bell saved over $150 million using forecasting and employee scheduling quantitative analysis models
- NBC television increased revenues by over $200 million by using quantitative analysis to develop better sales plans for advertisers
- Continental Airlines saved over $40 million every year using quantitative analysis models to quickly recover from weather delays and other disruptions

What is Quantitative Analysis? (1 of 4)

Quantitative analysis is a scientific approach to managerial decision making in which raw data are processed and manipulated to produce meaningful information

What is Quantitative Analysis? (2 of 4)

- **Quantitative factors** are data that can be accurately calculated
  - Different investment alternatives
  - Interest rates
  - Financial ratios
  - Cash flows and rates of return
  - Flow of materials through a supply chain

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What is Quantitative Analysis? (3 of 4)

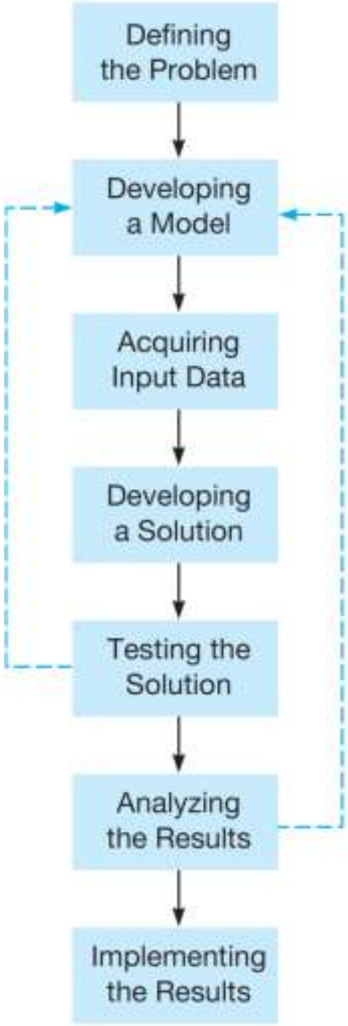
- **Qualitative factors** are more difficult to quantify but affect the decision process
    - The weather
    - State and federal legislation
    - Technological breakthroughs
    - The outcome of an election

What is Quantitative Analysis? (4 of 4)

- Quantitative and qualitative factors may have different roles
- Decisions based on quantitative data can be *automated*
- Generally quantitative analysis will *aid* the decision-making process
- Important in many areas of management
  - Production/Operations Management
  - Supply Chain Management
  - Business Analytics

The Quantitative Analysis Approach

FIGURE 1.1 The Quantitative Analysis Approach

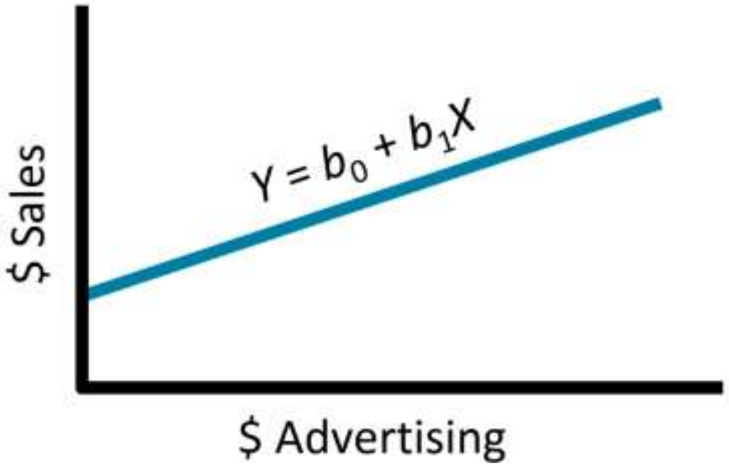


Defining the Problem

- Develop a clear and concise statement of the problem to provide direction and meaning
  - This may be the most important and difficult step
  - Go beyond symptoms and identify true causes
  - Concentrate on only a few of the problems – selecting the right problems is very important
  - *Specific* and *measurable* objectives may have to be developed

Developing a Model (1 of 2)

- Models are realistic, solvable, and understandable mathematical representations of a situation



- Different types of models

Physical models

Scale models

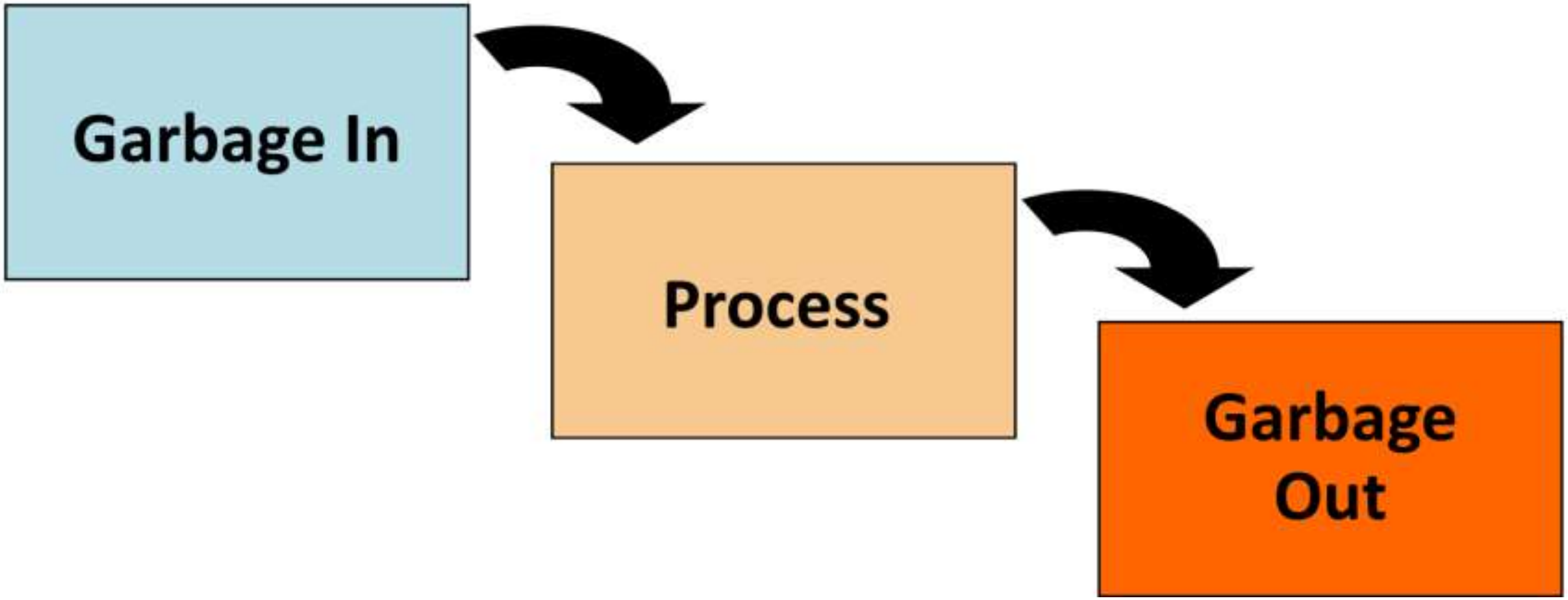
Schematic models

Developing a Model (2 of 2)

- Mathematical model – a set of mathematical relationships
- Models generally contain **variables** and **parameters**
  - *Controllable* variables, *decision* variables, are generally unknown
    - How many items should be ordered for inventory?
  - Parameters are known quantities that are a part of the model
    - What is the cost of placing an order?
- Required **input data** must be available

Acquiring Input Data

- Input data must be accurate – GIGO rule
- Data may come from a variety of sources – company reports, documents, employee interviews, direct measurement, or statistical sampling



Garbage In

Process

Garbage Out

Developing a Solution

- Manipulating the model to arrive at the best (optimal) solution
- Common techniques are
  - Solving equations
  - *Trial and error* – trying various approaches and picking the best result
  - *Complete enumeration* – trying all possible values
  - Using an **algorithm** – a series of repeating steps to reach a solution

Testing the Solution

- Both input data and the model should be tested for accuracy and completeness before analysis and implementation
  - New data can be collected to test the model
  - Results should be logical, consistent, and represent the real situation

Analyzing the Results

- Determine the implications of the solution
    - Implementing results often requires change in an organization
    - The impact of actions or changes needs to be studied and understood before implementation
- **Sensitivity analysis**, *postoptimality analysis*, determines how much the results will change if the model or input data changes
    - Sensitive models should be very thoroughly tested

Implementing the Results

- Implementation incorporates the solution into the company
  - Implementation can be very difficult
  - People may be resistant to changes
  - Many quantitative analysis efforts have failed because a good, workable solution was not properly implemented
- Changes occur over time, so even successful implementations must be monitored to determine if modifications are necessary

Modeling in the Real World

- Quantitative analysis models are used extensively by real organizations to solve real problems
  - In the real world, quantitative analysis models can be complex, expensive, and difficult to sell
  - Following the steps in the process is an important component of success

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How to Develop a Quantitative Analysis Model (1 of 3)

A mathematical model of profit:

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

- Revenue and expenses can be expressed in different ways

How to Develop a Quantitative Analysis Model (2 of 3)

Profit = Revenue − (Fixed cost + Variable cost)

Profit = (Selling price per unit)(Number of units sold) − [Fixed cost + (Variable costs per unit)(Number of units sold)]

Profit = $sX - [f + vX]$

Profit = $sX - f - vX$

where

s = selling price per unit

v = variable cost per unit

f = fixed cost

X = number of units sold

How to Develop a Quantitative Analysis Model (3 of 3)

Profit = Revenue − (F

Profit = (Selling price

[Fixed cost +
units sold)]

Profit = $sX - [f + vX]$

Profit = $sX - f - vX$

The *parameters* of this model are *f*, *v*, and *s* as these are the inputs inherent in the model.

The *decision variable* of interest is *X*.

where

s = selling price per unit v = variable cost per unit

f = fixed cost X = number of units sold

Pritchett's Precious Time Pieces (1 of 3)

- The company buys, sells, and repairs old clocks
  - Rebuilt springs sell for \$8 per unit
  - Fixed cost of equipment to build springs is \$1,000
  - Variable cost for spring material is \$3 per unit

$$s = 8 \qquad f = 1{,}000 \qquad v = 3$$

Number of spring sets sold = X

Profits = \$8X − \$1,000 − \$3X

If sales = 0, profits = − f = **− \$1,000**

If sales = 1,000, profits = [(\$8)(1,000) − \$1,000 − (\$3)(1,000)]

= \$4,000

Pritchett's Precious Time Pieces (2 of 3)

- Companies are often interested in the **break-even point** (BEP), the BEP is the number of units sold that will result in $0 profit

$$0 = sX - f - vX, \qquad \text{or} \qquad 0 = (s - v)X - f$$

Solving for *X*, we have

$$f = (s - v)X$$

$$X = f \div (s - v)$$

$$\text{BEP} = \frac{\text{Fixed cost}}{(\text{Selling price per unit}) - (\text{Variable cost per unit})}$$

Pritchett's Precious Time Pieces (3 of 3)

BEP for Pritchett's Precious Time Pieces

$$\text{BEP} = \$1{,}000 \div (\$8 - \$3) = 200 \text{ units}$$

- Sales of less than 200 units of rebuilt springs will result in a loss
- Sales of over 200 units of rebuilt springs will result in a profit

Advantages of Mathematical Modeling

1. Models can accurately represent reality.
2. Models can help a decision maker formulate problems.
3. Models can give us insight and information.
4. Models can save time and money in decision making and problem solving.
5. A model may be the only way to solve large or complex problems in a timely fashion.
6. A model can be used to communicate problems and solutions to others.

Models Categorized by Risk

- Mathematical models that do not involve risk or chance are called **deterministic models**
  - All of the values used in the model are known with complete certainty
- Mathematical models that involve risk or chance are called **probabilistic models**
  - Values used in the model are estimates based on probabilities

Possible Problems in the Quantitative Analysis Approach (1 of 2)

- Defining the problem
  - Problems may not be easily identified
  - Conflicting viewpoints
  - Impact on other departments
  - Beginning assumptions
  - Solution outdated
- Developing a model
  - Fitting the textbook models
  - Understanding the model

Possible Problems in the Quantitative Analysis Approach (2 of 2)

- Acquiring accurate input data
  - Using accounting data
  - Validity of the data
- Developing a solution
  - Hard-to-understand mathematics
  - Only one answer is limiting
- Testing the solution
  - Solutions not always intuitively obvious
- Analyzing the results
  - How will it affect the total organization

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Implementation – Not Just the Final Step (1 of 2)

- Lack of commitment and resistance to change
  - Fear formal analysis processes will reduce management's decision-making power
  - Fear previous intuitive decisions exposed as inadequate
  - Uncomfortable with new thinking patterns
  - Action-oriented managers may want "quick and dirty" techniques
  - Management support and user involvement are important

Implementation – Not Just the Final Step (2 of 2)

- Lack of commitment by quantitative analysts
  - Analysts should be involved with the problem and care about the solution
  - Analysts should work with users and take their feelings into account

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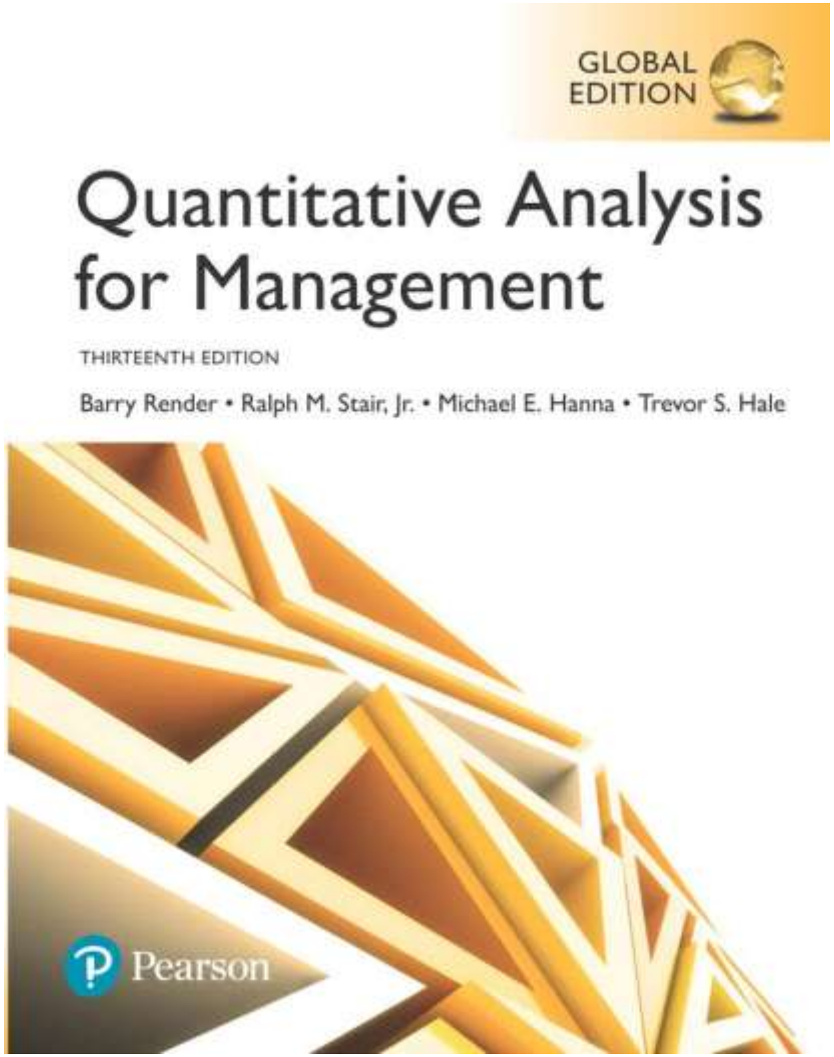


Chapter 3

Decision Analysis

Introduction

- The success or failure that a person experiences in life depend on the decisions that he or she makes.
- What is involved in making a good decision?
- Decision theory is an analytic and systematic approach to the study of decision making.
- A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here.

The Six Steps in Decision Making

1. Clearly define the problem at hand
2. List the possible alternatives
3. Identify the possible outcomes or states of nature
4. List the payoff (typically profit) of each combination of alternatives and outcomes
5. Select one of the mathematical decision theory models
6. Apply the model and make your decision

Thompson Lumber Company (1 of 3)

- **Step 1** – Define the problem
    - The company is Considering expanding by manufacturing and marketing a new product – backyard storage sheds
- **Step 2** – List alternatives
    - Construct a large new plant
    - Construct a small new plant
    - Do not develop the new product line
- **Step 3** – Identify possible outcomes, states of nature
    - The market could be favorable or unfavorable

Thompson Lumber Company (2 of 3)

- **Step 4** – List the payoffs
  - Identify **conditional values** for the profits for large plant, small plant, and no development for the two possible market conditions
- **Step 5** – Select the decision model
  - Depends on the environment and amount of risk and uncertainty
- **Step 6** – Apply the model to the data

Thompson Lumber Company (3 of 3)

TABLE 3.1 Decision Table with Conditional Values for Thompson Lumber

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	200,000	−180,000
Construct a small plant	100,000	−20,000
Do nothing	0	0

Note: It is important to include all alternatives, including “do nothing.”

Types of Decision-Making Environments

- **Decision making under certainty**
  - The decision maker knows with certainty the consequences of every alternative or decision choice
- **Decision making under uncertainty**
  - The decision maker does not know the probabilities of the various outcomes
- **Decision making under risk**
  - The decision maker knows the probabilities of the various outcomes

Decision Making Under Uncertainty

- Criteria for making decisions under uncertainty
  1. Maximax (optimistic)
  2. Maximin (pessimistic)
  3. Criterion of realism (Hurwicz)
  4. Equally likely (Laplace)
  5. Minimax regret

Optimistic

- Used to find the alternative that maximizes the maximum payoff – **maximax** criterion
  - Locate the maximum payoff for each alternative
  - Select the alternative with the maximum number

TABLE 3.2 Thompson's Maximax Decision

	STATE OF NATURE		
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**	**MAXIMUM IN A ROW ($)**
Construct a large plant	200,000	−180,000	200,000 Maximax
Construct a small plant	100,000	−20,000	100,000
Do nothing	0	0	0

Pessimistic

- Used to find the alternative that maximizes the minimum payoff – **maximin** criterion
  - Locate the minimum payoff for each alternative
  - Select the alternative with the maximum number

TABLE 3.3 Thompson's Maximin Decision

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	200,000	−180,000	−180,000
Construct a small plant	100,000	−20,000	−20,000
Do nothing	0	0	0

Maximin

Criterion of Realism (Hurwicz) (1 of 2)

- Often called **weighted average**
  - Compromise between optimism and pessimism
  - Select a coefficient of realism α, with $0 \le \alpha \le 1$

    $\alpha = 1$ is perfectly optimistic

    $\alpha = 0$ is perfectly pessimistic
  - Compute the weighted averages for each alternative
  - Select the alternative with the highest value

$$\text{Weighted average} = \alpha(\text{best in row}) + (1-\alpha)(\text{worst in row})$$

Criterion of Realism (Hurwicz) (2 of 2)

For the large plant alternative using $\alpha = 0.8$

$$(0.8)(200{,}000) + (1-0.8)(-180{,}000) = 124{,}000$$

For the small plant alternative using $\alpha = 0.8$

$$(0.8)(100{,}000) + (1-0.8)(-20{,}000) = 76{,}000$$

TABLE 3.4 Thompson’s Criterion of Realism Decision

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	CRITERION OF REALISM OR WEIGHTED AVERAGE ($\alpha = 0.8$) ($)
Construct a large plant	200,000	−180,000	124,000 Realism
Construct a small plant	100,000	−20,000	76,000
Do nothing	0	0	0

Equally Likely (Laplace)

- Considers all the payoffs for each alternative
  - Find the average payoff for each alternative
  - Select the alternative with the highest average

TABLE 3.5 Thompson's Equally Likely Decision

	STATE OF NATURE		
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**	**ROW AVERAGE ($)**
Construct a large plant	200,000	−180,000	10,000
Construct a small plant	100,000	−20,000	40,000 Equally likely
Do nothing	0	0	0

Minimax Regret (1 of 4)

- Based on **opportunity loss** or **regret**
  - The difference between the optimal profit and actual payoff for a decision
    1. Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative
    2. Calculate opportunity loss by subtracting each payoff in the column from the best payoff in the column
    3. Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number

Minimax Regret (2 of 4)

TABLE 3.6 Determining Opportunity Losses for Thompson Lumber

STATE OF NATURE	
FAVORABLE MARKET ($)	**UNFAVORABLE MARKET ($)**
200,000 − 200,000	0 − (−180,000)
200,000 − 100,000	0 − (−20,000)
200,000 − 0	0 − 0

Minimax Regret (3 of 4)

TABLE 3.7 Opportunity Loss Table for Thompson Lumber

	STATE OF NATURE	
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Minimax Regret (4 of 4)

TABLE 3.8 Thompson's Minimax Decision Using Opportunity Loss

	STATE OF NATURE		
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**	**MAXIMUM IN A ROW ($)**
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
			Minimax
Do nothing	200,000	0	200,000

Decision Making Under Risk (1 of 2)

- When there are several possible states of nature and the probabilities associated with each possible state are known
  - Most popular method – choose the alternative with the highest expected monetary value (EMV)

$$\text{EMV}\left(\text{alternative}\right) = \sum X_i P\left(X_i\right)$$

where

X_i = payoff for the alternative in state of nature i

$P(X_i)$ = probability of achieving payoff X_i (i.e., probability of state of nature i)

$\sum$ = summation symbol

Decision Making Under Risk (2 of 2)

- Expanding the equation

EMV (alternative *i*) = (payoff of first state of nature)
× (probability of first state of nature)
+ (payoff of second state of nature)
× (probability of second state of nature)
+ … + (payoff of last state of nature)
× (probability of last state of nature)

EMV for Thompson Lumber (1 of 2)

- Each market outcome has a probability of occurrence of 0.50
- Which alternative would give the highest EMV?

EMV (large plant) = ($200,000)(0.5) + (−$180,000)(0.5)
= $10,000
EMV (small plant) = ($100,000)(0.5) + (−$20,000)(0.5)
= $40,000
EMV (do nothing) = ($0)(0.5) + ($0)(0.5)
= $0

EMV for Thompson Lumber (2 of 2)

TABLE 3.9 Decision Table with Probabilities and EMVs for Thompson Lumber

	STATE OF NATURE		
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**	**EMV ($)**
Construct a large plant	200,000	−180,000	10,000
Construct a small plant	100,000	−20,000	40,000 Best EMV
Do nothing	0	0	0
Probabilities	0.50	0.50	

Expected Value of Perfect Information (EVPI) (1 of 6)

- **EVPI** places an upper bound on what you should pay for additional information
- **EVwPI** is the long run average return if we have perfect information before a decision is made

$$\text{EVwPI} = \sum(\text{best payoff in state of nature } i)\,(\text{probability of state of nature } i)$$

Expected Value of Perfect Information (EVPI) (2 of 6)

- Expanded EVwPI becomes

EVwPI = (best payoff for first state of nature)
× (probability of first state of nature)
+ (best payoff for second state of nature)
× (probability of second state of nature)
+ … + (best payoff for last state of nature)
× (probability of last state of nature)

And

EVPI = EVwPI – Best EMV

Expected Value of Perfect Information (EVPI) (3 of 6)

- Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable)
- Additional information will cost $65,000
- Should Thompson Lumber purchase the information?

Expected Value of Perfect Information (EVPI) (4 of 6)

TABLE 3.10 Decision Table with Perfect Information

	STATE OF NATURE		
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	EMV ($)
Construct a large plant	200,000	−180,000	10,000
Construct a small plant	100,000	−20,000	40,000
Do nothing	0	0	0
With perfect information	200,000	0	100,000
			EVwPI
Probabilities	0.50	0.50	

Expected Value of Perfect Information (EVPI) (5 of 6)

- The maximum EMV without additional information is \$40,000
  - Therefore

$$\begin{aligned} \text{EVPI} &= \text{EVwPI} - \text{Maximum EMV} \\ &= \$100{,}000 - \$40{,}000 \\ &= \$60{,}000 \end{aligned}$$

So the maximum Thompson should pay for the additional information is \$60,000

Expected Value of Perfect Information (EVPI) (6 of 6)

- The maximum EMV without additional information is $40,000
  - Therefore

Thompson should not pay $65,000 for this information

$$\begin{aligned} \text{EVPI} &= \text{EVwPI} - \text{Maximum EMV} \\ &= \$100{,}000 - \$40{,}000 \\ &= \$60{,}000 \end{aligned}$$

So the maximum Thompson should pay for the additional information is $60,000

Expected Opportunity Loss (1 of 2)

- Expected opportunity loss (EOL) is the cost of not picking the best solution
  - Construct an opportunity loss table
  - For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together
  - Minimum EOL will always result in the same decision as maximum EMV
  - Minimum EOL will always equal EVPI

Expected Opportunity Loss (2 of 2)

EOL (large plant) = (0.50)($0) + (0.50)($180,000) = $90,000

EOL (small plant) = (0.50)($100,000) + (0.50)($20,000) = $60,000

EOL (do nothing) = (0.50)($200,000) + (0.50)($0) = $100,000

TABLE 3.11 EOL Table for Thompson Lumber

	STATE OF NATURE		
ALTERNATIVE	**FAVORABLE MARKET ($)**	**UNFAVORABLE MARKET ($)**	**EOL ($)**
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	60,000
			Best EOL
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

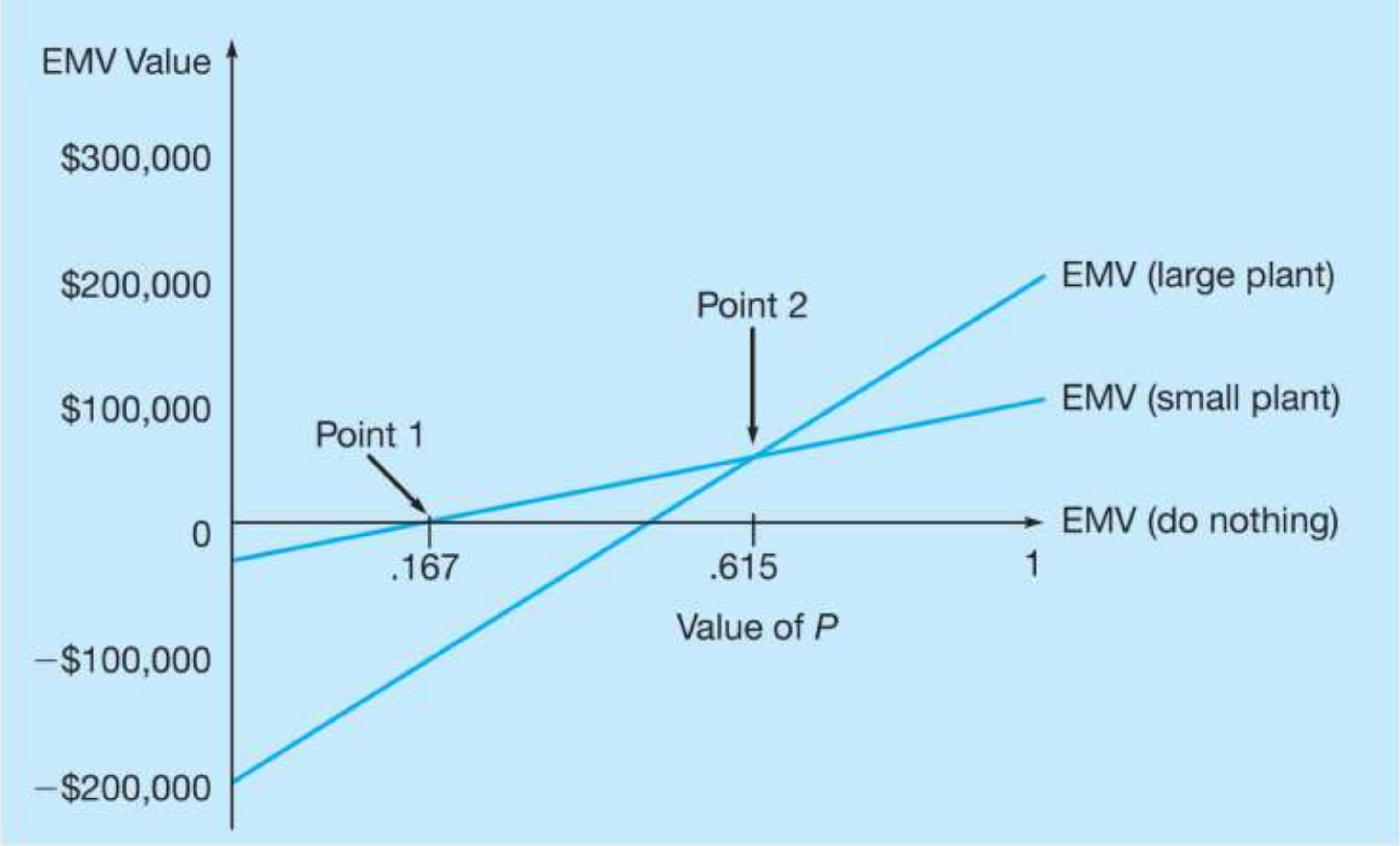
Sensitivity Analysis (1 of 4)

Define P = probability of a favorable market

$$\begin{aligned}
\text{EMV(large plant)} &= \$200{,}000P - \$180{,}000)(1 - P) \\
&= \$200{,}000P - \$180{,}000 + \$180{,}000P \\
&= \$380{,}000P - \$180{,}000 \\
\text{EMV(small plant)} &= \$100{,}000P - \$20{,}000)(1 - P) \\
&= \$100{,}000P - \$20{,}000 + \$20{,}000P \\
&= \$120{,}000P - \$20{,}000 \\
\text{EMV(do nothing)} &= \$0P + 0(1 - P) \\
&= \$0
\end{aligned}$$

Sensitivity Analysis (2 of 4)

FIGURE 3.1 Sensitivity Analysis



Sensitivity Analysis (3 of 4)

Point 1: EMV(do nothing) = EMV(small plant)

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

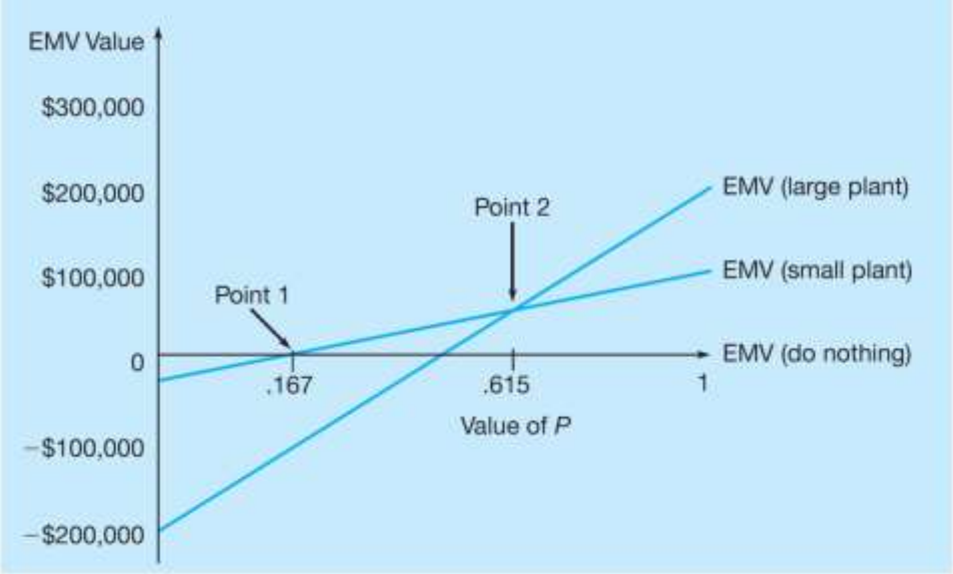
Point 2: EMV(small plant) = EMV(large plant)

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

Sensitivity Analysis (4 of 4)

FIGURE 3.1 Sensitivity Analysis



BEST ALTERNATIVE	RANGE OF *P* VALUES
Do nothing	Less than 0.167
Construct a small plant	0.167 – 0.615
Construct a large plant	Greater than 0.615

A Minimization Example (1 of 8)

- Three year lease for a copy machine
  - Which machine should be selected?

TABLE 3.12 Payoff Table with Monthly Copy Costs for Business Analytics Department

	10,000 COPIES PER MONTH	**20,000 COPIES PER MONTH**	**30,000 COPIES PER MONTH**
Machine A	950	1,050	1,150
Machine B	850	1,100	1,350
Machine C	700	1,000	1,300

A Minimization Example (2 of 8)

- Three year lease for a copy machine
  - Which machine should be selected?

TABLE 3.13 Best and Worst Payoffs (Costs) for Business Analytics Department

	10,000 COPIES PER MONTH	20,000 COPIES PER MONTH	30,000 COPIES PER MONTH	BEST PAYOFF (MINIMUM)	WORST PAYOFF (MAXIMUM)
Machine A	950	1,050	1,150	950	1,150
Machine B	850	1,100	1,350	850	1,350
Machine C	700	1,000	1,300	700	1,300

A Minimization Example (3 of 8)

- Using Hurwicz criteria with 70% coefficient

Weighted average = 0.7(best payoff)
+ (1 – 0.7)(worst payoff)

For each machine
Machine A: 0.7(950) + 0.3(1,150) = 1,010
Machine B: 0.7(850) + 0.3(1,350) = 1,000
Machine C: 0.7(700) + 0.3(1,300) = 880

A Minimization Example (4 of 8)

- For equally likely criteria

For each machine

Machine A: (950 + 1,050 + 1,150)÷3 = 1,050

Machine B: (850 + 1,100 + 1,350)÷3 = 1,100

Machine C: (700 + 1,000 + 1,300)÷3 = 1,000

A Minimization Example (5 of 8)

- For EMV criteria

USAGE	PROBABILITY
10,000	0.40
20,000	0.30
30,000	0.30

A Minimization Example (6 of 8)

- For EMV criteria

TABLE 3.14 Expected Monetary Values and Expected Values with Perfect Information for Business Analytics Department

	10,000 COPIES PER MONTH	20,000 COPIES PER MONTH	30,000 COPIES PER MONTH	EMV
Machine A	950	1,050	1,150	1,040
Machine B	850	1,100	1,350	1,075
Machine C	700	1,000	1,300	970
With perfect information	700	1,000	1,150	925
Probability	0.4	0.3	0.3	

A Minimization Example (7 of 8)

- For EVPI

ected

lytics

EVwPI = $925

Best EMV without perfect information = $970

EVPI = 970 − 925 = $45

				EMV
Machine A	950	1,050	1,150	1,040
Machine B	850	1,100	1,350	1,075
Machine C	700	1,000	1,300	970
With perfect information	700	1,000	1,150	925
Probability	0.4	0.3	0.3	

Pearson

A Minimization Example (8 of 8)

- Opportunity loss criteria

TABLE 3.15 Opportunity Loss Table for Business Analytics Department

	10,000 COPIES PER MONTH	20,000 COPIES PER MONTH	30,000 COPIES PER MONTH	MAXIMUM	EOL
Machine A	250	50	0	250	115
Machine B	150	100	200	200	150
Machine C	0	0	150	150	45
Probability	0.4	0.3	0.3		

Decision Trees

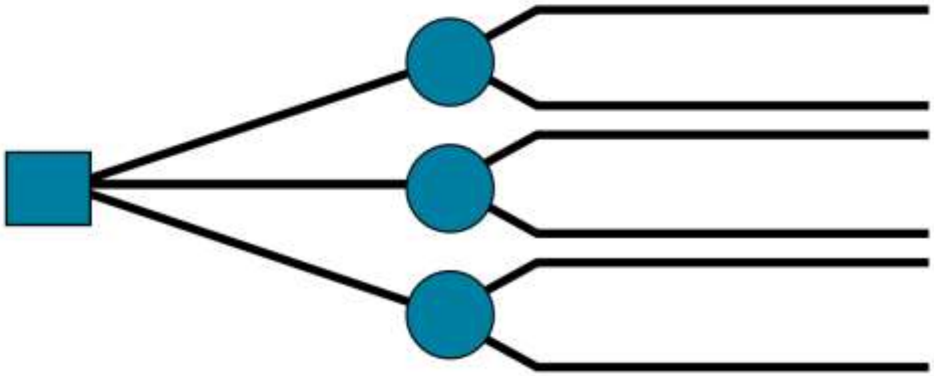
- Any problem that can be presented in a decision table can be graphically represented in a **decision tree**
  - Most beneficial when a sequence of decisions must be made
  - All decision trees contain **decision points/nodes** and **state-of-nature points/nodes**
  - At decision nodes one of several alternatives may be chosen
  - At state-of-nature nodes one state of nature will occur

Five Steps of Decision Tree Analysis

1. Define the problem
2. Structure or draw the decision tree
3. Assign probabilities to the states of nature
4. Estimate payoffs for each possible combination of alternatives and states of nature
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node

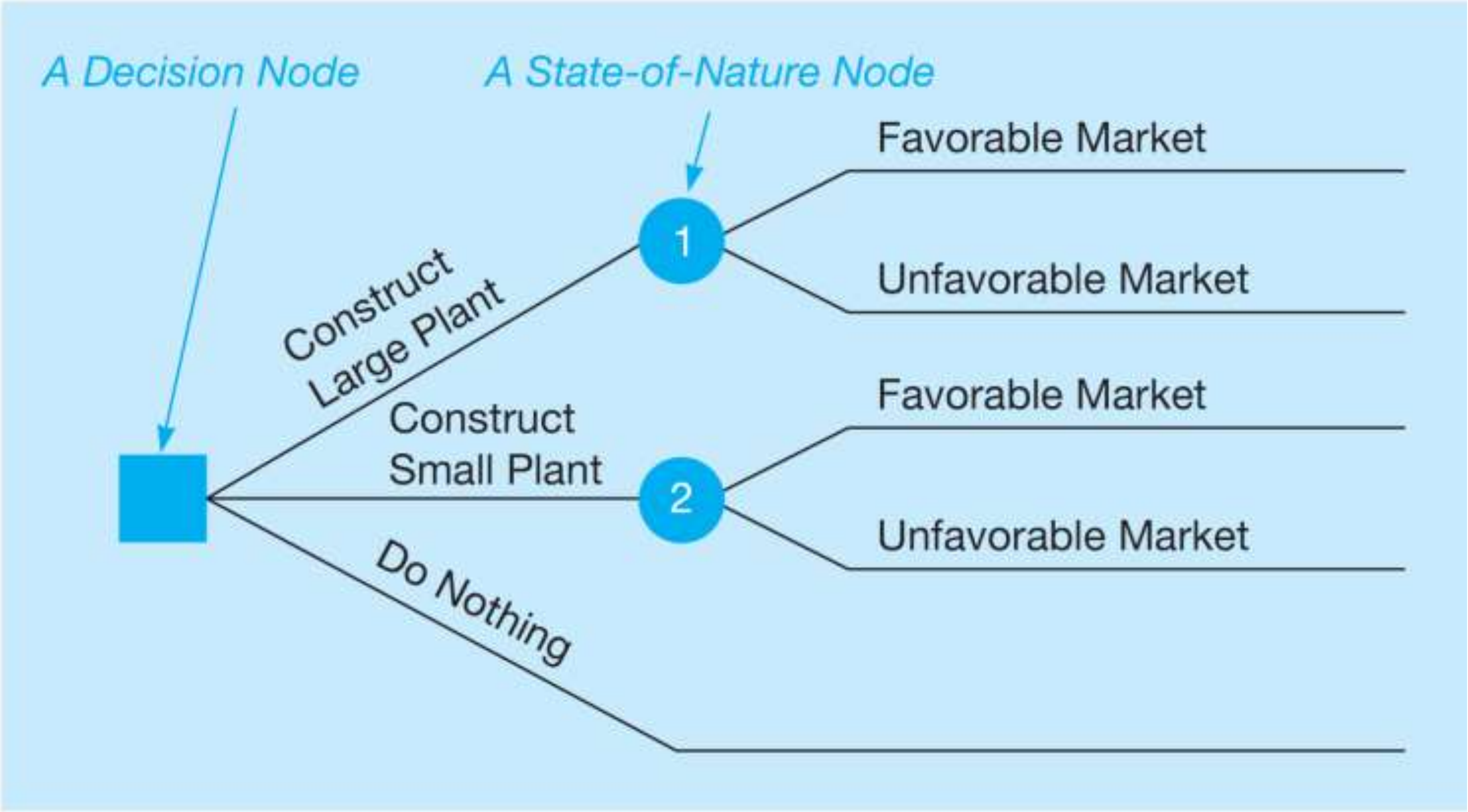
Structure of Decision Trees

- Trees start from left to right
- Trees represent decisions and outcomes in sequential order
- Squares represent decision nodes
- Circles represent states of nature nodes
- Lines or branches connect the decisions nodes and the states of nature



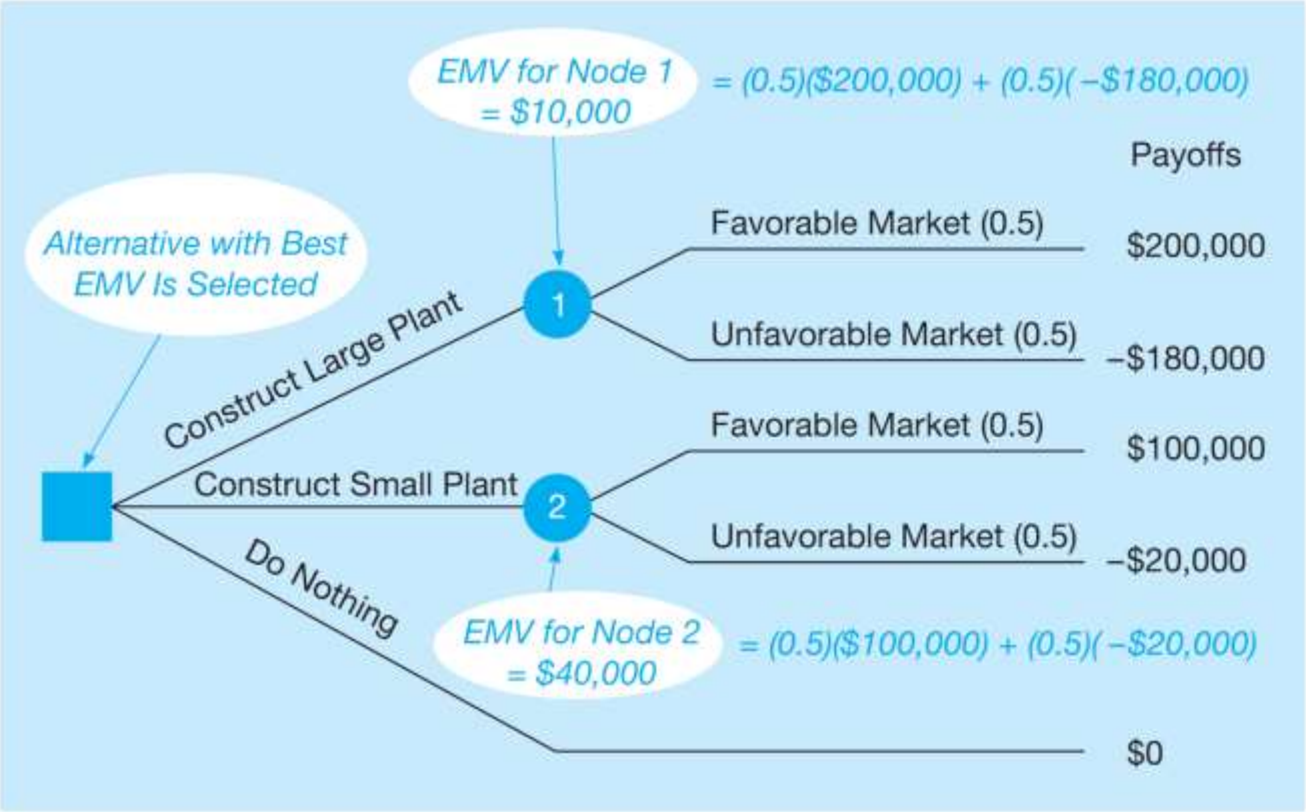
Thompson’s Decision Tree (1 of 2)

FIGURE 3.2 Thompson’s Decision Tree



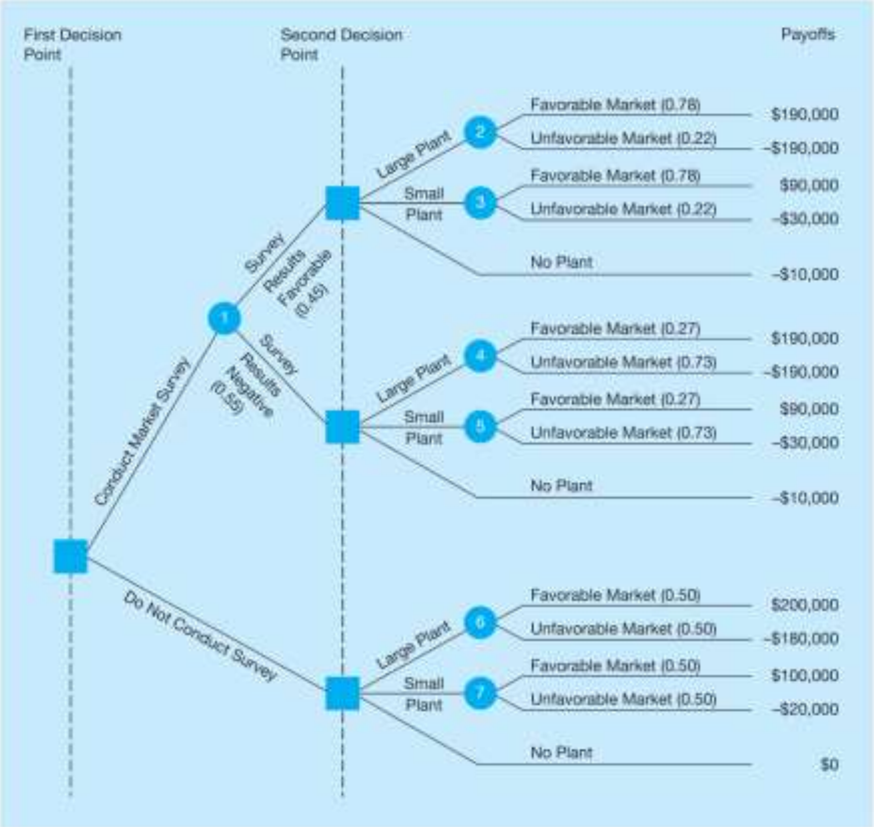
Thompson's Decision Tree (2 of 2)

FIGURE 3.3 Completed and Solved Decision Tree for Thompson Lumber



Thompson’s Complex Decision Tree (1 of 5)

FIGURE 3.4 Larger Decision Tree with Payoffs and Probabilities for Thompson Lumber



Thompson's Complex Decision Tree (2 of 5)

1. Given favorable survey results

EMV(node 2) = EMV(large plant | positive survey)
= (0.78)($190,000) + (0.22)(−$190,000)
= $106,400

EMV(node 3) = EMV(small plant | positive survey)
= (0.78)($90,000) + (0.22)(−$30,000)
= $63,600

EMV for no plant = −$10,000

Thompson’s Complex Decision Tree (3 of 5)

2. Given negative survey results

EMV(node 4) = EMV(large plant | negative survey)
= (0.27)($190,000) + (0.73)(−$190,000)
= −$87,400

EMV(node 5) = EMV(small plant | negative survey)
= (0.27)($90,000) + (0.73)(−$30,000)
= $2,400

EMV for no plant = −$10,000

Thompson's Complex Decision Tree

3. Compute the expected value of the market survey,

   EMV(node 1) = EMV(conduct survey)
   = (0.45)($106,400) + (0.55)($2,400)
   = $47,880 + $1,320 = $49,200

4. If the market survey is not conducted,

   EMV(node 6) = EMV(large plant)
   = (0.50)($200,000) + (0.50)(–$180,000) = $10,000
   EMV(node 7) = EMV(small plant)
   = (0.50)($100,000) + (0.50)(–$20,000) = $40,000
   EMV for no plant = $0

5. The best choice is to seek marketing information.

Thompson's Complex Decision Tree

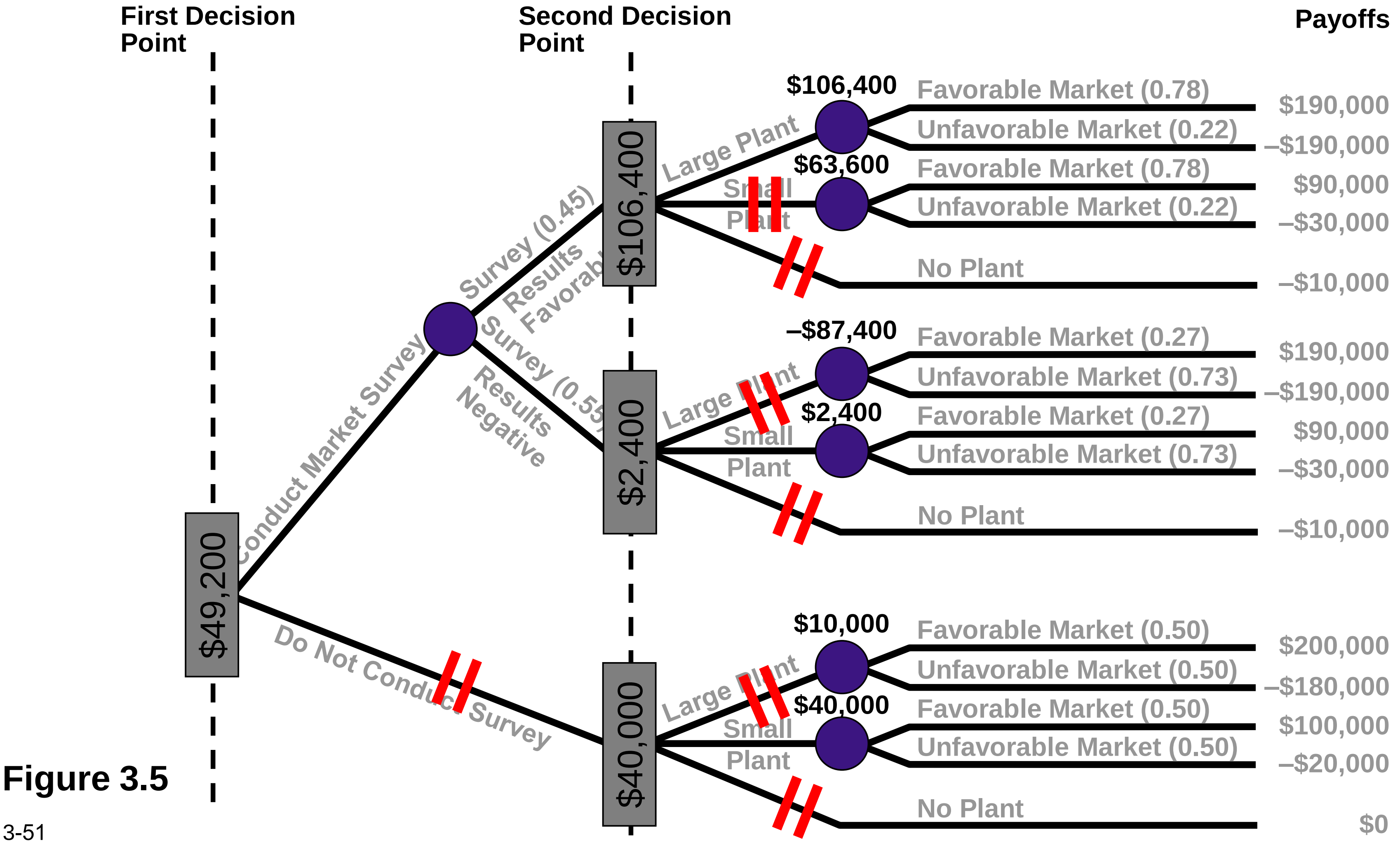


Figure 3.5

Expected Value of Sample Information

- Suppose Thompson wants to know the actual value of doing the survey.

$$\textbf{EVSI} = \left(\begin{array}{c}\text{Expected value}\\ \textit{with}\text{ sample}\\ \text{information, assuming}\\ \text{no cost to gather it}\end{array}\right) - \left(\begin{array}{c}\text{Expected value}\\ \text{of best decision}\\ \textit{without}\text{ sample}\\ \text{information}\end{array}\right)$$

= (EV with sample information + cost)
– (EV without sample information)

EVSI = ($49,200 + $10,000) – $40,000 = $19,200

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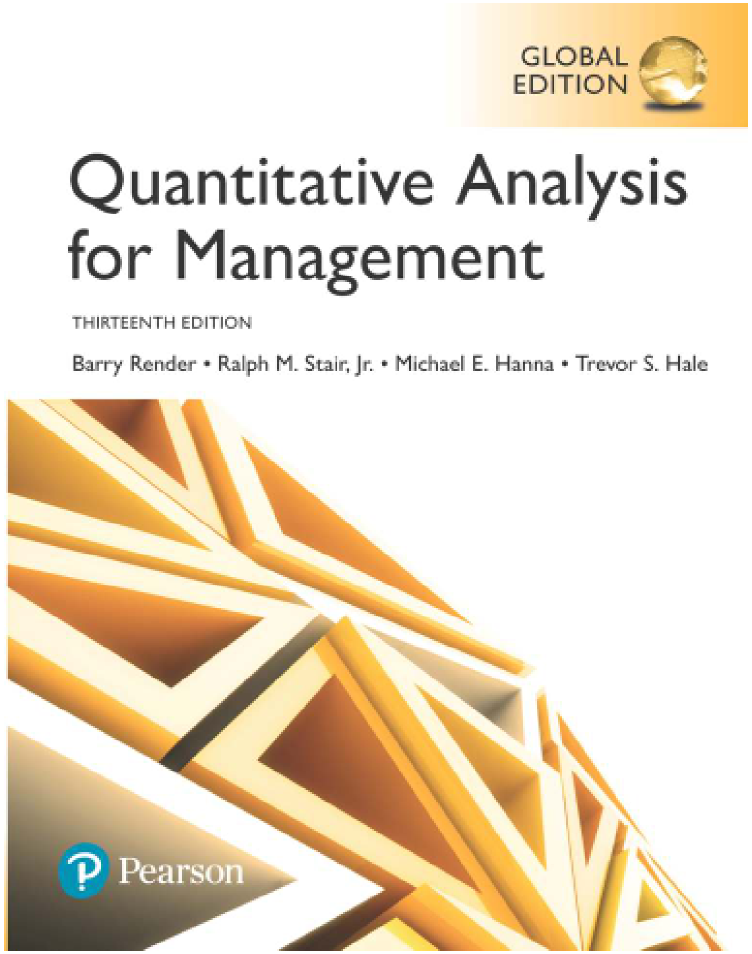

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Chapter 5

Forecasting

What is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities

Principles of Forecasting

Many types of forecasting models that differ in complexity and amount of data they use, and the way they generate forecasts:

1. Forecasts are rarely perfect
2. Forecasts are more accurate for groups or family of items than for individual items
3. Forecast are more accurate for shorter than longer time periods

Types of Forecasting Models

- Qualitative methods often called-judgmental methods
  - Are methods in which the forecast is made subjectively by the forecaster
  - They are educated guesses by forecasters or experts based on intuition, knowledge, and experience.
- Quantitative methods – based on mathematical modeling:
  - Are methods in which the forecast is based on mathematical modeling

Quantitative Methods

- **Quantitative methods can be divided into two categories:**
- **Time Series Models:**
  - Assumes information needed to generate a forecast is contained in a time series of data
  - Time series: is a series of observations taken at a regular intervals over a specified period of time
  - Assumes the future will follow same patterns as the past
- **Causal Models or Associative Models**
  - Explores cause-and-effect relationships
  - Sales volume and advertising

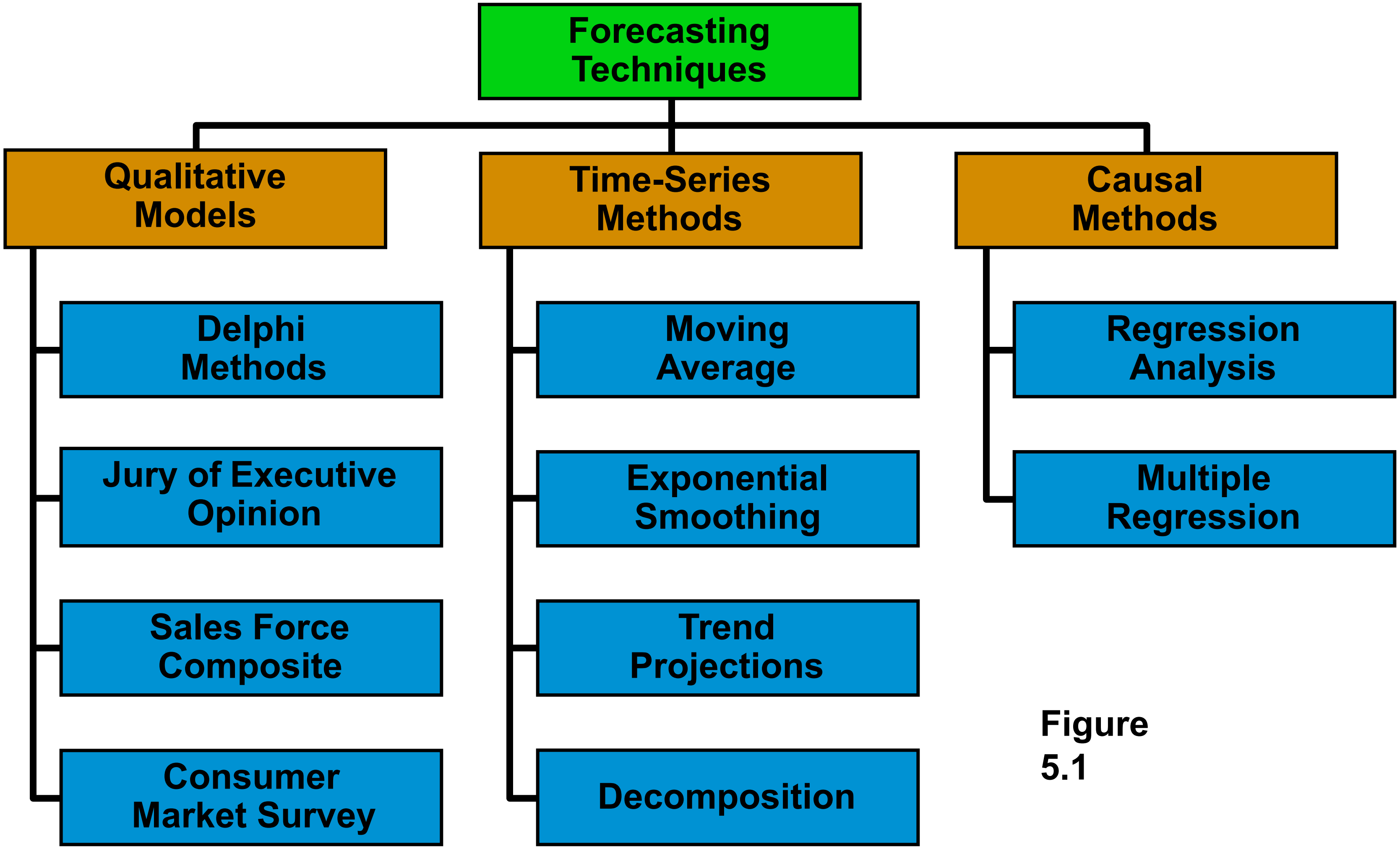
Introduction

- Managers are always trying to reduce uncertainty and make better estimates of what will happen in the future.
  - This is the main purpose of forecasting.
  - Some firms use subjective methods: intuition, experience, guessing.
  - There are also several quantitative techniques, including:
    - Moving averages
    - Exponential smoothing
    - Trend projections
    - Least squares regression analysis

Introduction

- Eight steps to forecasting:
  1. Determine the use of the forecast—what objective are we trying to obtain?
  2. Select the items or quantities that are to be forecasted.
  3. Determine the time horizon of the forecast.
  4. Select the forecasting model or models.
  5. Gather the data needed to make the forecast.
  6. Validate the forecasting model.
  7. Make the forecast.
  8. Implement the results.

Forecasting Models


Forecasting Techniques

Qualitative Models

- Delphi Methods
- Jury of Executive Opinion
- Sales Force Composite
- Consumer Market Survey

Time-Series Methods

- Moving Average
- Exponential Smoothing
- Trend Projections
- Decomposition

Causal Methods

- Regression Analysis
- Multiple Regression

Figure 5.1

Qualitative Models

- *Qualitative models* incorporate judgmental or subjective factors.
- These are useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.
- Common qualitative techniques are:
  - Delphi method.
  - Jury of executive opinion.
  - Sales force composite (estimates).
  - Consumer market surveys.

Qualitative Models

- ***Delphi Method*** **– This is an iterative (correlated) group process where (possibly geographically dispersed)** ***respondents*** **provide input to** ***decision makers.***
- ***Jury of Executive Opinion*** **– This method collects opinions of a small group of high-level managers, possibly using statistical models for analysis.**
- ***Sales Force Composite (estimates)*** **– This allows individual salespersons estimate the sales in their region and the data is compiled at a district or national level.**
- ***Consumer Market Survey*** **– Input is solicited from customers or potential customers regarding their purchasing plans.**

Quantitative Models

- *Time-series models* **attempt to predict the future based on the past.**
- **Common time-series models are:**
  - **Moving average.**
  - **Exponential smoothing.**
  - **Trend projections.**
  - **Decomposition (dividing).**
- **Regression analysis is used in trend projections and one type of decomposition model.**

Quantitative Models

- *Causal models* use variables or factors that might influence the quantity being forecasted.
- The objective is to build a model with the best statistical relationship between the variable being forecast and the independent variables.
- Regression analysis is the most common technique used in causal modeling.

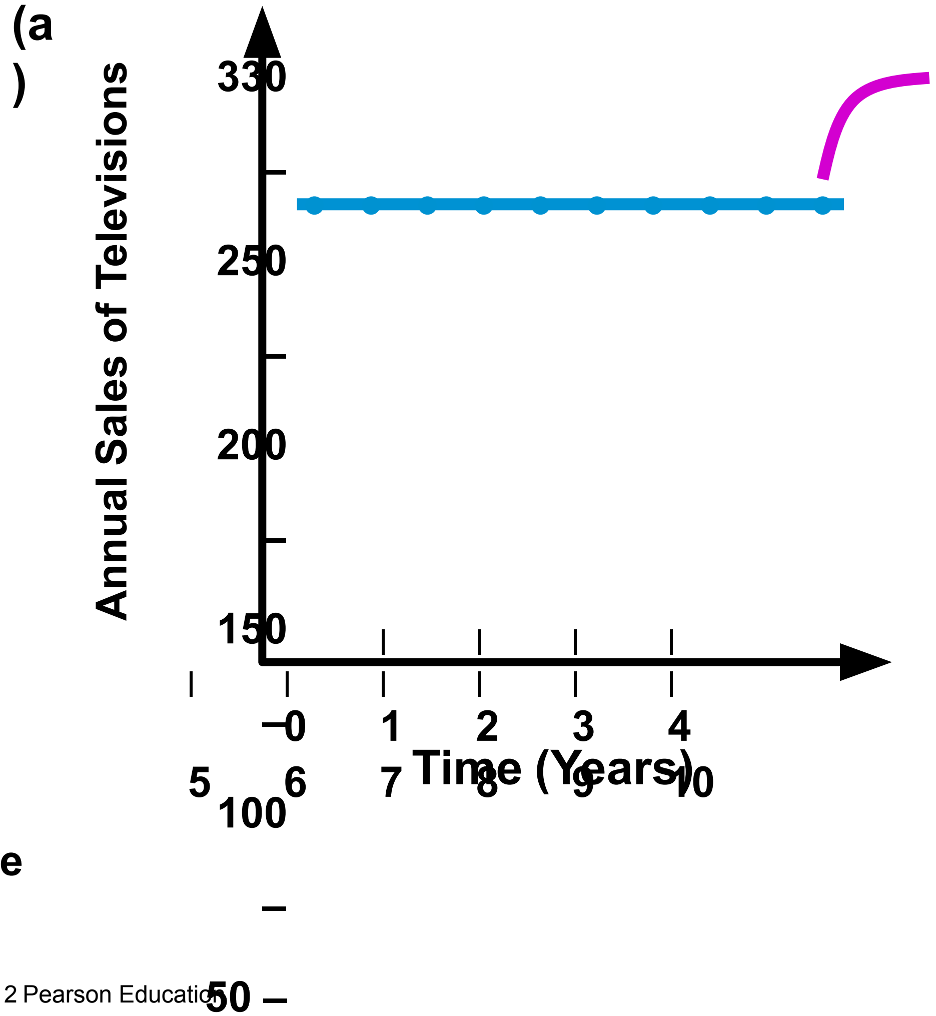
Scatter Diagrams

Wacker Distributors wants to forecast sales for three different products (annual sales in the table, in units):

YEAR	TELEVISION SETS	RADIOS	COMPACT DISC PLAYERS
1	250	300	110
2	250	310	100
3	250	320	120
4	250	330	140
5	250	340	170
6	250	350	150
7	250	360	160
8	250	370	190
9	250	380	200
10	250	390	190

Table 5.1

Scatter Diagram for TVs



- Sales appear to be constant over time

  Sales = 250

- A good estimate of sales in year 11 is 250 televisions

Figure 5.2a

Scatter Diagram for Radios

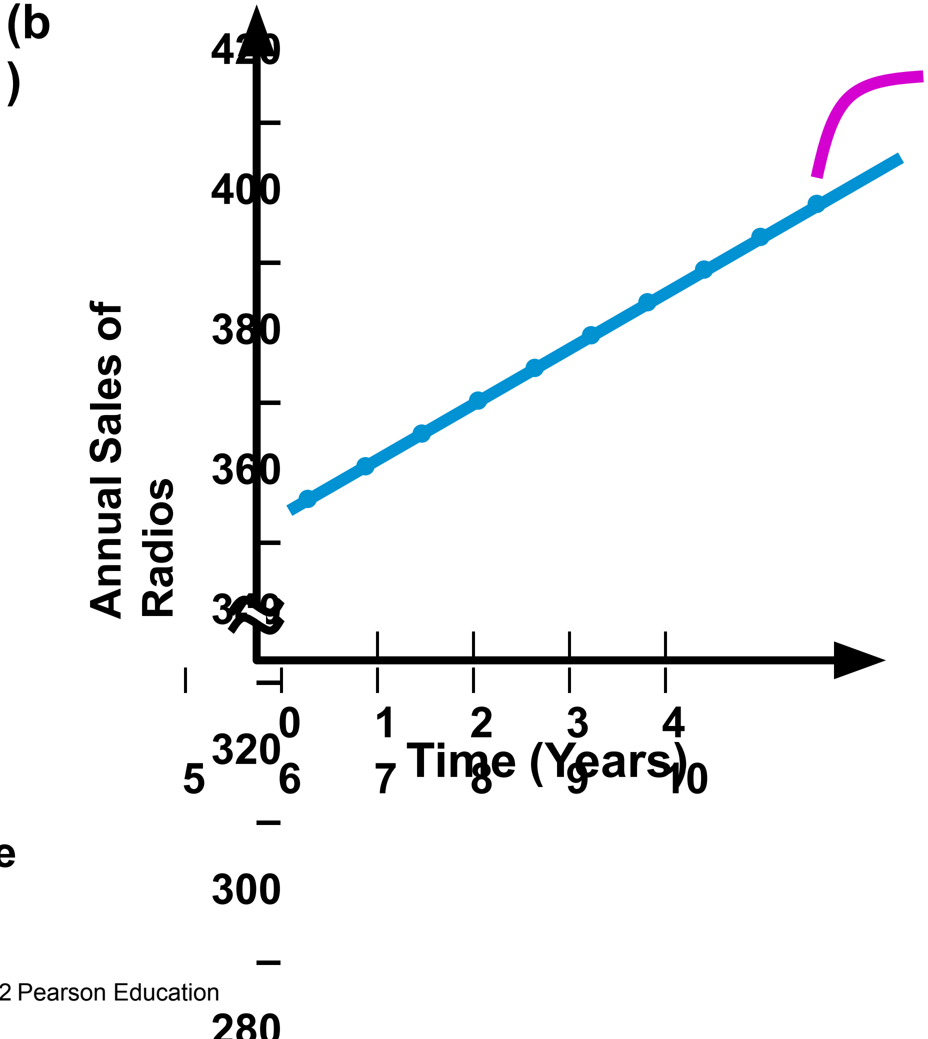


Figure 5.2b

- Sales appear to be increasing at a constant rate of 10 radios per year

Sales = 290 + 10(Year)

- A reasonable estimate of sales in year 11 is 400 radios.

Scatter Diagram for CD Players

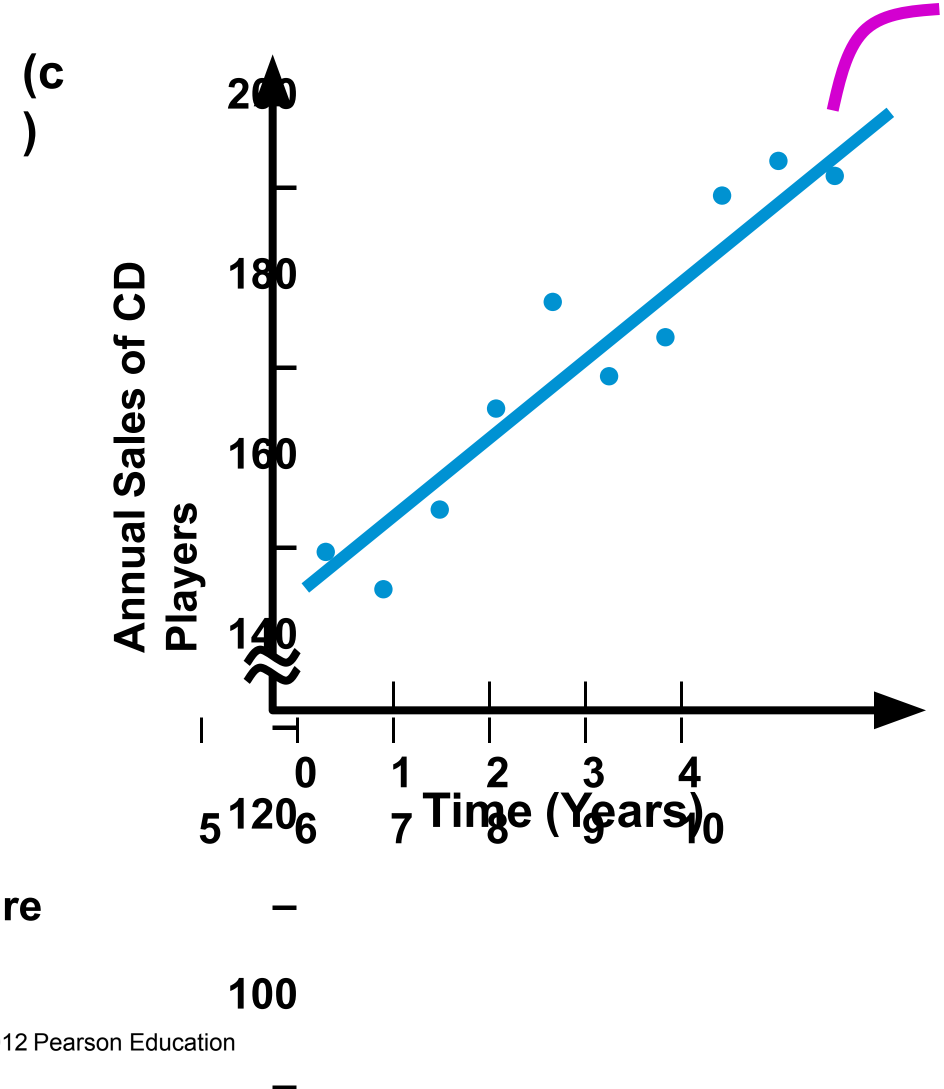


Figure 5.2c

- This trend line may not be perfectly accurate because of variation from year to year
- Sales appear to be increasing
- A forecast would probably be a larger figure each year

Measures of Forecast Accuracy

- **We compare forecasted values with actual values to see how well one model works or to compare models.**

Forecast error = Actual value – Forecast value

- **One measure of accuracy is the *mean absolute deviation* (*MAD*):**

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
2	100	110	|100 – 110| = 10
3	120	100	|120 – 110| = 20
4	140	120	|140 – 120| = 20
5	170	140	|170 – 140| = 30
6	150	170	|150 – 170| = 20
7	160	150	|160 – 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

Table 5.2

Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
			– 110| = 10
			– 110| = 20
			– 120| = 20
			– 140| = 30
			– 170| = 20
			– 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

Table 5.2

Measures of Forecast Accuracy

- There are other popular measures of forecast accuracy.
- The *mean squared error:*

$$\text{MSE} = \frac{\sum(\text{error})^2}{n}$$

- The *mean absolute percent error:*

$$\text{MAPE} = \frac{\sum\left|\frac{\text{error}}{\text{actual}}\right|}{n}100\%$$

- And *bias* is the average error.

Time-Series Forecasting Models

- A time series is a sequence of evenly spaced events.
- Time-series forecasts predict the future based solely on the past values of the variable, and other variables are ignored.

Components of a Time-Series

A time series typically has four components:

1. *Trend* (T) is the gradual upward or downward movement of the data over time.
2. *Seasonality* (S) is a pattern of demand fluctuations above or below the trend line that repeats at regular intervals.
3. *Cycles* (C) are patterns in annual data that occur every several years.
4. *Random variations* (R) are “blips” in the data caused by chance or unusual situations, and follow no discernible pattern.

Decomposition of a Time-Series

Product Demand Charted over 4 Years, with Trend and Seasonality Indicated

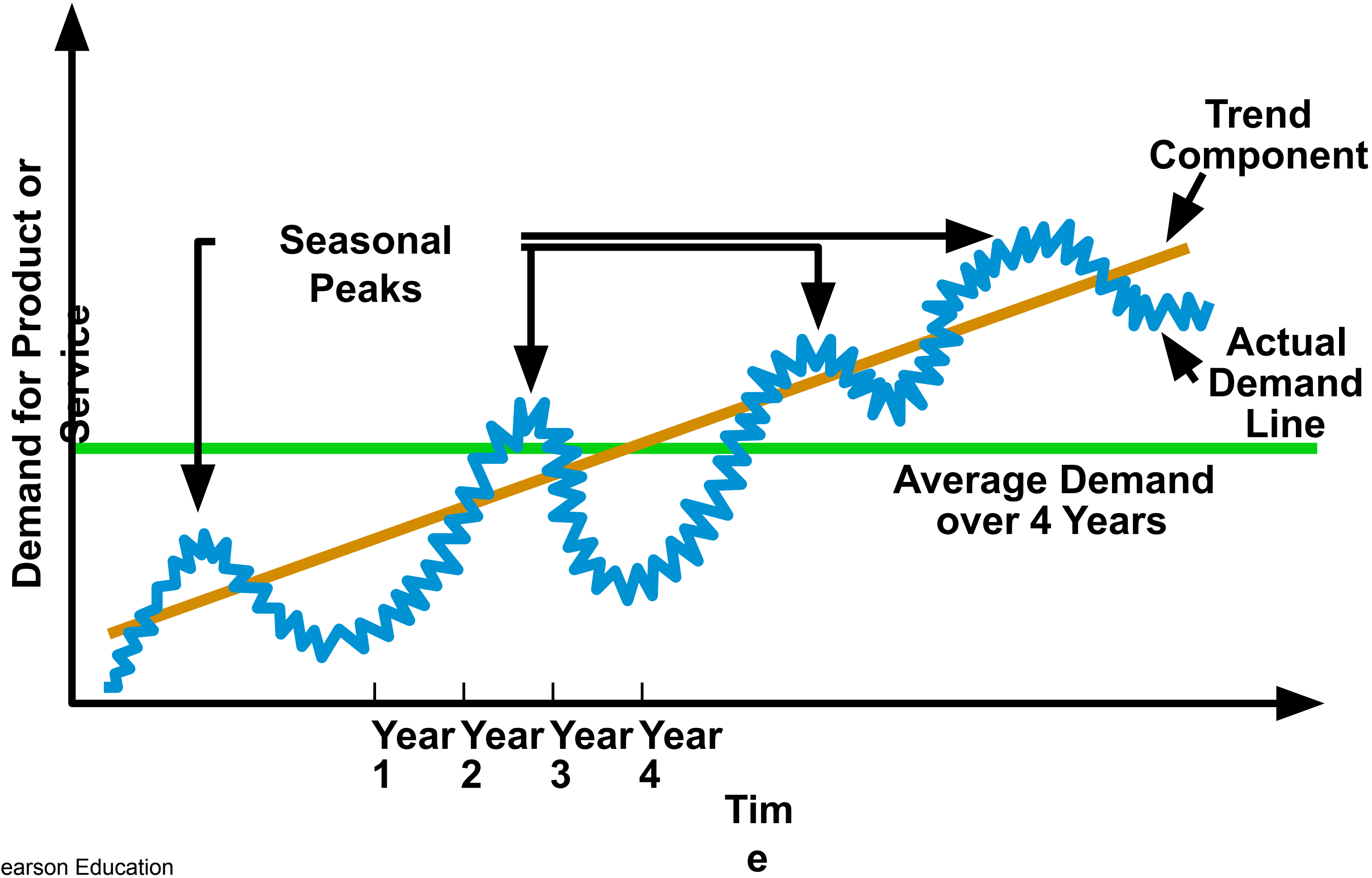


Figure 5.3

Moving Averages

- *Moving averages* can be used when demand is relatively steady over time.
- The next forecast is the average of the most recent n data values from the time series.
- This methods tends to smooth out short-term irregularities in the data series.

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

Moving Averages

- **Mathematically:**

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Where:

F_{t+1} = forecast for time period t + 1

Y_t = actual value in time period t

n = number of periods to average

Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed.
- They have collected data for the past year.
- They are using a three-month moving average to forecast demand (n = 3).

Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	(10 + 12 + 13)/3 = 11.67
May	19	(12 + 13 + 16)/3 = 13.67
June	23	(13 + 16 + 19)/3 = 16.00
July	26	(16 + 19 + 23)/3 = 19.33
August	30	(19 + 23 + 26)/3 = 22.67
September	28	(23 + 26 + 30)/3 = 26.33
October	18	(26 + 30 + 28)/3 = 28.00
November	16	(30 + 28 + 18)/3 = 25.33
December	14	(28 + 18 + 16)/3 = 20.67
January	—	(18 + 16 + 14)/3 = 16.00

Table 5.3

Weighted Moving Averages

- *Weighted moving averages* use weights to put more emphasis on previous periods.
- This is often used when a trend or other pattern is emerging.

$$F_{t+1} = \frac{\sum (\text{Weight in period } i)(\text{Actual value in period})}{\sum (\text{Weights})}$$

- Mathematically:

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

where

w_i = weight for the i^{th} observation

Wallace Garden Supply

- Wallace Garden Supply decides to try a weighted moving average model to forecast demand for its Storage Shed.
- They decide on the following weighting scheme:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of the weights

3 x Sales last month + 2 x Sales two months ago + 1 X Sales three months ago

Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 X 13) + (2 X 12) + (10)]/6 = 12.17
May	19	[(3 X 16) + (2 X 13) + (12)]/6 = 14.33
June	23	[(3 X 19) + (2 X 16) + (13)]/6 = 17.00
July	26	[(3 X 23) + (2 X 19) + (16)]/6 = 20.50
August	30	[(3 X 26) + (2 X 23) + (19)]/6 = 23.83
September	28	[(3 X 30) + (2 X 26) + (23)]/6 = 27.50
October	18	[(3 X 28) + (2 X 30) + (26)]/6 = 28.33
November	16	[(3 X 18) + (2 X 28) + (30)]/6 = 23.33
December	14	[(3 X 16) + (2 X 18) + (28)]/6 = 18.67
January	—	[(3 X 14) + (2 X 16) + (18)]/6 = 15.33

Table 5.4

Exponential Smoothing

- *Exponential smoothing* is a type of moving average that is easy to use and requires little record keeping of data.

New forecast = Last period's forecast
+ α(Last period's actual demand
– Last period's forecast)

Here α is a weight (or *smoothing constant*) in which $0 \leq \alpha \leq 1$.

Exponential Smoothing

Mathematically:

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

Where:

F_{t+1} = new forecast (for time period t + 1)
F_t = pervious forecast (for time period t)
α = smoothing constant ($0 \leq \alpha \leq 1$)
Y_t = pervious period's actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period.

Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142.
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

New forecast (for March demand) = 142 + 0.2(153 – 142)
= 144.2 or 144 autos

- If actual demand in March was 136 autos, the April forecast would be:

New forecast (for April demand) = 144.2 + 0.2(136 – 144.2)
= 142.6 or 143 autos

Selecting the Smoothing Constant

- Selecting the appropriate value for α is key to obtaining a good forecast.
- The objective is always to generate an accurate forecast.
- The general approach is to develop trial forecasts with different values of α and select the α that results in the lowest MAD.

Exponential Smoothing

Port of Baltimore Exponential Smoothing Forecast for *α=0.1 and α=0.5.*

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING α =0.10	FORECAST USING α =0.50
1	180	175	175
2	168	175.5 = 175.00 + 0.10(180 – 175)	177.5
3	159	174.75 = 175.50 + 0.10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + 0.10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + 0.10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + 0.10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + 0.10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + 0.10(180 – 178.02)	186.30
9	?	178.60 = 178.22 + 0.10(182 – 178.22)	184.15

Table 5.5

Exponential Smoothing

Absolute Deviations and MADs for the Port of Baltimore Example

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH α = 0.10	ABSOLUTE DEVIATIONS FOR α = 0.10	FORECAST WITH α = 0.50	ABSOLUTE DEVIATIONS FOR α = 0.50
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	$\boxed{29.98}$	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.3

Best

Table 5.6

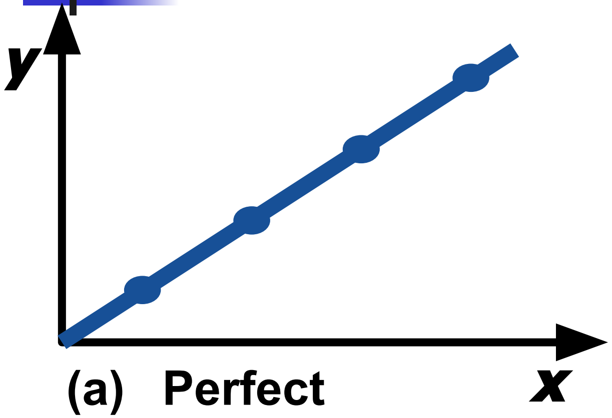
Correlation

- How strong is the linear relationship between the variables?
- Correlation does not necessarily imply causality!
- Coefficient of correlation, r , measures degree of association
  - Values range from -1 to +1

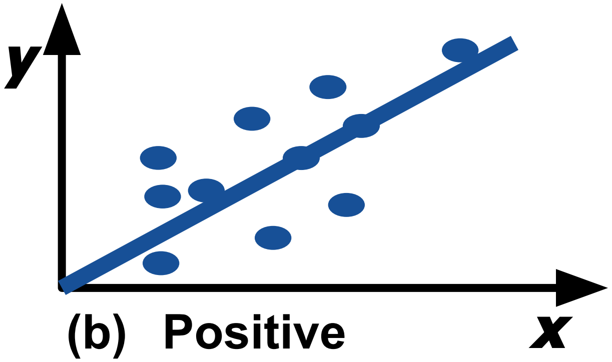
Correlation Coefficient

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

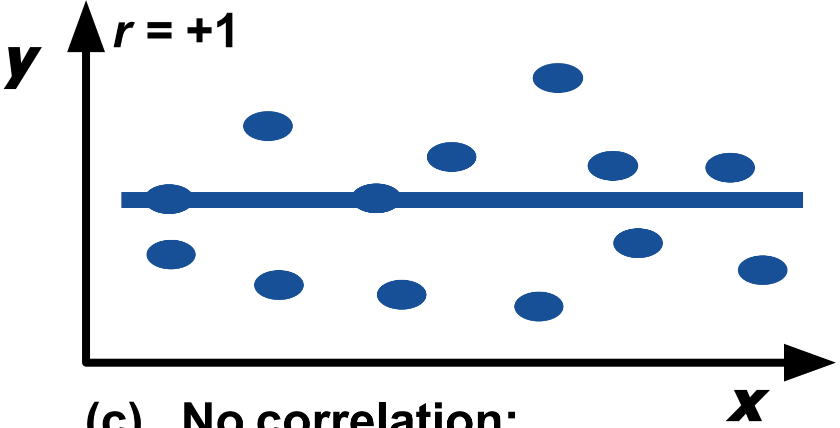
Correlation Coefficient



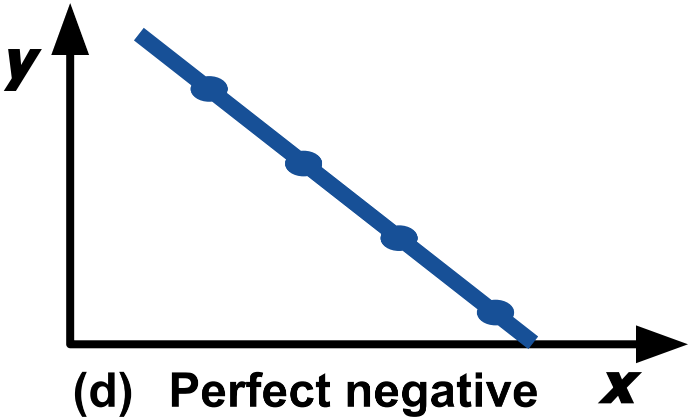
(a) Perfect positive correlation: $r = +1$



(b) Positive correlation: $0 < r < 1$



(c) No correlation: $r = 0$



(d) Perfect negative correlation: $r = -1$

Correlation

- Coefficient of Determination, r^2, measures the percent of change in y predicted by the change in x
  - Values range from 0 to 1
  - Easy to interpret

For the Nodel Construction example:

$\boldsymbol{r = .901}$

$\boldsymbol{r^2 = .81}$

Common Measures of Error

Mean Absolute Deviation (MAD)

$$MAD = \frac{\Sigma\ |\text{Actual - Forecast}|}{n}$$

Mean Squared Error (MSE)

$$MSE = \frac{\Sigma\ (\text{Forecast Errors})^2}{n}$$

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with α = .10	Absolute Deviation for α = .10	Rounded Forecast with α = .50	Absolute Deviation for α = .50
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

st Error

bsolute Deviation

= .50

$$MAD = \frac{\Sigma\,|\text{deviations}|}{n}$$

For α = .10 $= 82.45/8 = 10.31$

For α = .50 $= 98.62/8 = 12.33$

8	182	178.22	3.78	186.30	4.30
			82.45	98.62	

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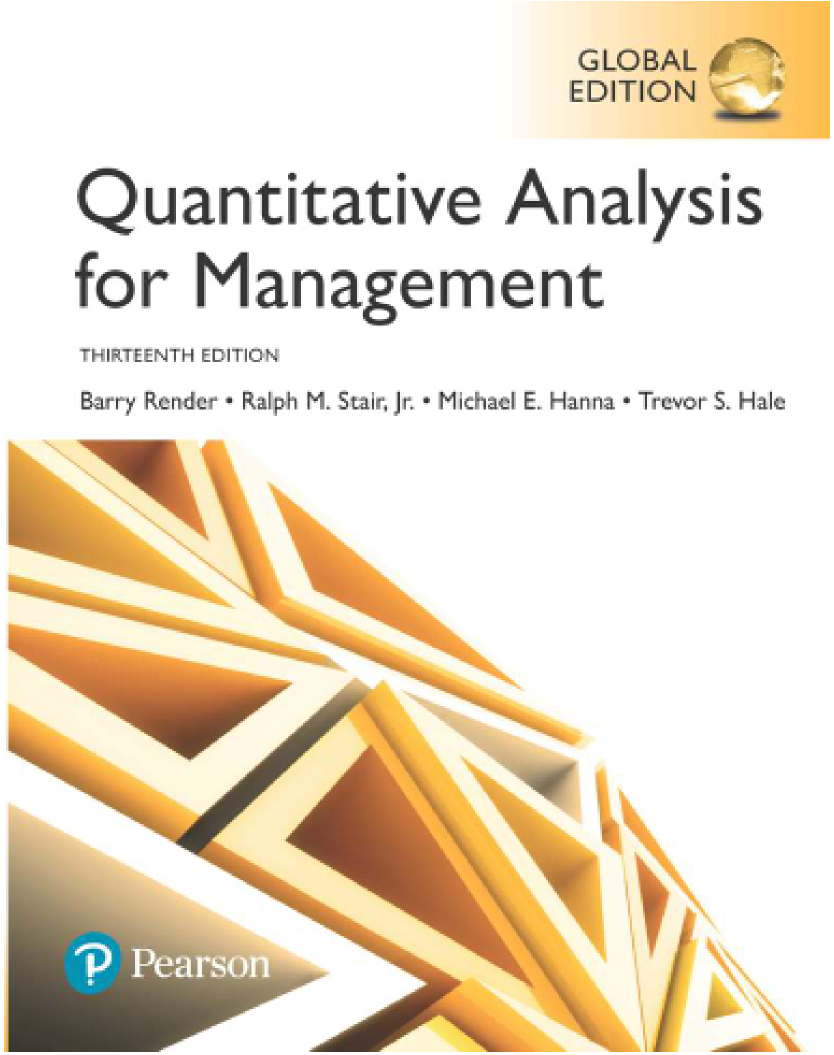

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Chapter 7

Linear Programming Models: Graphical and Computer Methods

Introduction

- Many management decisions involve making the most effective use of limited resources
- **Linear programming (LP)**
  - Widely used mathematical modeling technique
  - Planning and decision making relative to resource allocation
- Broader field of **mathematical programming**
  - Here programming refers to modeling and solving a problem mathematically

Requirements of a Linear Programming Problem

- LP has been applied in many areas over the past 50 years
- All LP problems have 4 properties in common
  1. All problems seek to *maximize* or *minimize* some quantity (the *objective function*)
  2. The presence of restrictions or *constraints* that limit the degree to which we can pursue our objective
  3. There must be alternative courses of action to choose from
  4. The objective and constraints in problems must be expressed in terms of *linear* equations or *inequalities*

Basic Assumptions of LP

- We assume conditions of *certainty* exist and numbers in the objective and constraints are known with certainty and do not change during the period being studied
- We assume *proportionality* exists in the objective and constraints
- We assume *additivity* in that the total of all activities equals the sum of the individual activities
- We assume *divisibility* in that solutions need not be whole numbers
- All answers or variables are *nonnegative*

LP Properties and Assumptions

PROPERTIES OF LINEAR PROGRAMS
1. One objective function
2. One or more constraints
3. Alternative courses of action
4. Objective function and constraints are linear
ASSUMPTIONS OF LP
1. Certainty
2. Proportionality
3. Additivity
4. Divisibility
5. Nonnegative variables

Table 7.1

Formulating LP Problems (1 of 2)

- Developing a mathematical model to represent the managerial problem
- Steps in formulating a LP problem
  1. Completely understand the managerial problem being faced
  2. Identify the objective and the constraints
  3. Define the decision variables
  4. Use the decision variables to write mathematical expressions for the objective function and the constraints

Formulating LP Problems (2 of 2)

- Common LP application – **product mix problem**
- Two or more products are produced using limited resources
- Maximize profit based on the profit contribution per unit of each product
- Determine how many units of each product to produce

Flair Furniture Company (1 of 6)

- Flair Furniture produces inexpensive tables and chairs
- Processes are similar, both require carpentry work and painting and varnishing
    - Each table takes 4 hours of carpentry and 2 hours of painting and varnishing
    - Each chair requires 3 of carpentry and 1 hour of painting and varnishing
    - There are 240 hours of carpentry time available and 100 hours of painting and varnishing
    - Each table yields a profit of $70 and each chair a profit of $50

Flair Furniture Company (2 of 6)

- The company wants to determine the best combination of tables and chairs to produce to reach the maximum profit

TABLE 7.2 Flair Furniture Company Data

	HOURS REQUIRED TO PRODUCE 1 UNIT		
DEPARTMENT	TABLES (T)	CHAIRS (C)	AVAILABLE HOURS THIS WEEK
Carpentry	4	3	240
Painting and varnishing	2	1	100
Profit per unit	$70	$50	

Flair Furniture Company (3 of 6)

- The objective is

  Maximize profit

- The constraints are
  1. The hours of carpentry time used cannot exceed 240 hours per week
  2. The hours of painting and varnishing time used cannot exceed 100 hours per week
- The **decision variables** are

  T = number of tables to be produced per week

  C = number of chairs to be produced per week

Flair Furniture Company (4 of 6)

- Create objective function in terms of T and C

  $$\text{Maximize profit} = \$70T + \$50C$$

- Develop mathematical relationships for the two constraints
  - For carpentry, total time used is

    (4 hours per table)(Number of tables produced)

    + (3 hours per chair)(Number of chairs produced)
  - First constraint is

    Carpentry time used ≤ Carpentry time available

    $$4T + 3C \le 240 \text{ (hours of carpentry time)}$$

Flair Furniture Company (5 of 6)

- Similarly

Painting and varnishing time used
≤ Painting and varnishing time available

$2T + 1C \le 100$ (hours of painting and varnishing time)

This means that each table produced requires two hours of painting and varnishing time

  – Both of these constraints restrict production capacity and affect total profit

Flair Furniture Company (6 of 6)

- The values for T and C must be nonnegative

$T \geq 0$ (number of tables produced is greater than or equal to 0)
$C \geq 0$ (number of chairs produced is greater than or equal to 0)

The complete problem stated mathematically

$$\text{Maximize profit} = \$70T + \$50C$$

subject to

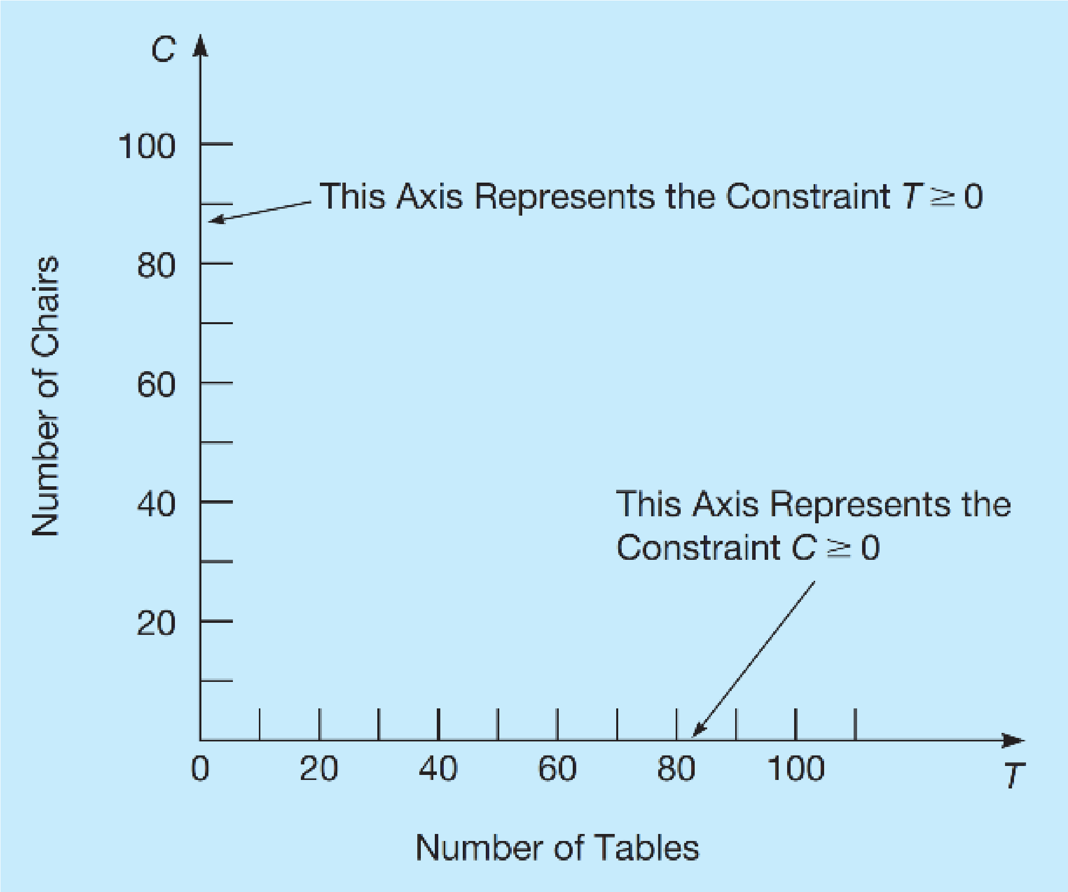
$4T + 3C \leq 240$ (carpentry constraint)
$2T + 1C \leq 100$ (painting and varnishing constraint)
$T, C \geq 0$ (nonnegativity constraint)

Graphical Solution to an LP Problem

- Easiest way to solve a small LP problems is graphically
- Only works when there are just two decision variables
  - Not possible to plot a solution for more than two variables
- Provides valuable insight into how other approaches work
- **Nonnegativity constraints** mean that we are always working in the first (or northeast) quadrant of a graph

Graphical Representation of Constraints (1 of 11)

FIGURE 7.1 Quadrant Containing All Positive Values



Graphical Representation of Constraints (2 of 11)

- The first step is to identify a set or region of feasible solutions
- Plot each constraint equation on a graph
- Graph the equality portion of the constraint equations

$$4T + 3C = 240$$

- Solve for the axis intercepts and draw the line

Graphical Representation of Constraints (3 of 11)

- When Flair produces no tables, the carpentry constraint is:

$$4(0) + 3C = 240$$

$$3C = 240$$

$$C = 80$$

- Similarly for no chairs:

$$4T + 3(0) = 240$$

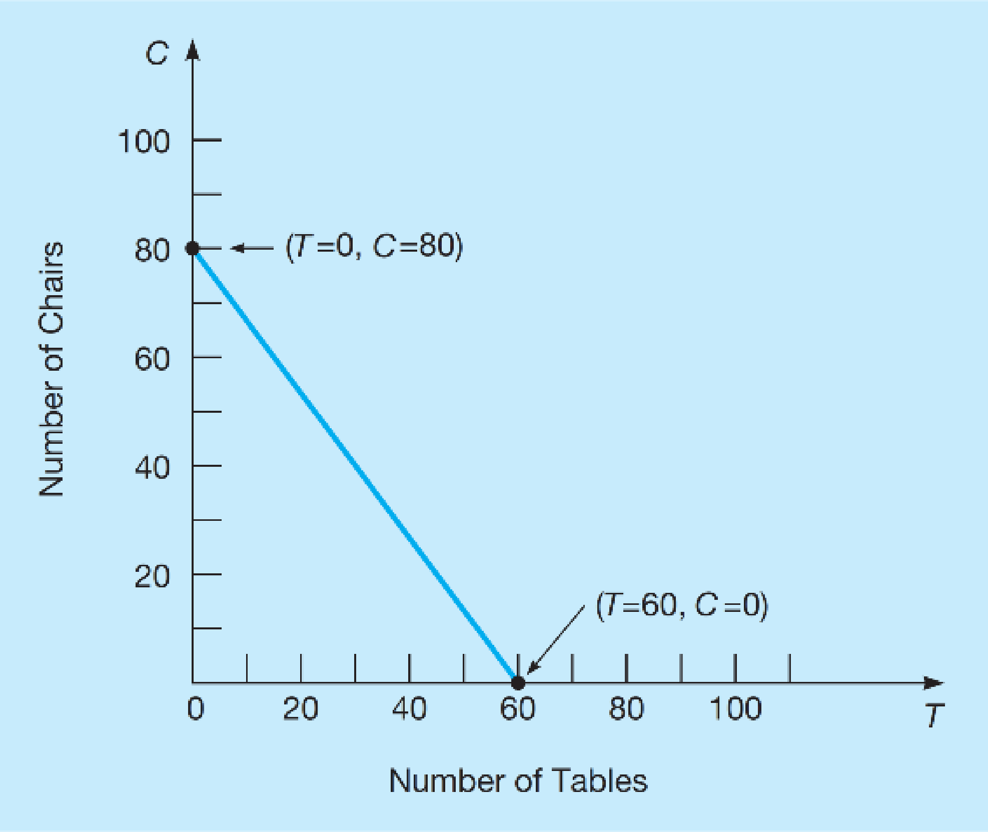
$$4T = 240$$

$$T = 60$$

  - This line is shown on the following graph

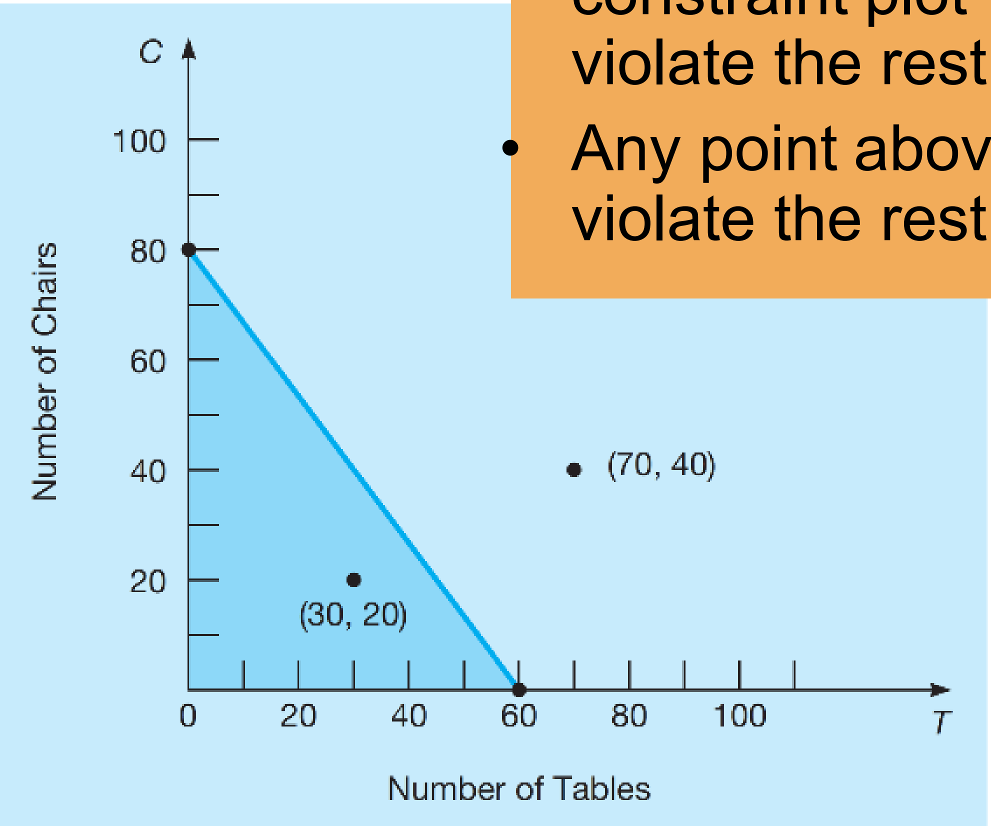
Graphical Representation of Constraints (4 of 11)

FIGURE 7.2 Graph of Carpentry Constraint Equation 4T + 3C = 240



Graphical Representation of Constraints (5 of 11)

FIGURE 7.3 Region that Satisfies the Carpentry Constraint



- Any point on or below the constraint plot will not violate the restriction
- Any point above the plot will violate the restriction

Graphical Representation of Constraints (6 of 11)

- The point (30, 20) lies below the line and satisfies the constraint

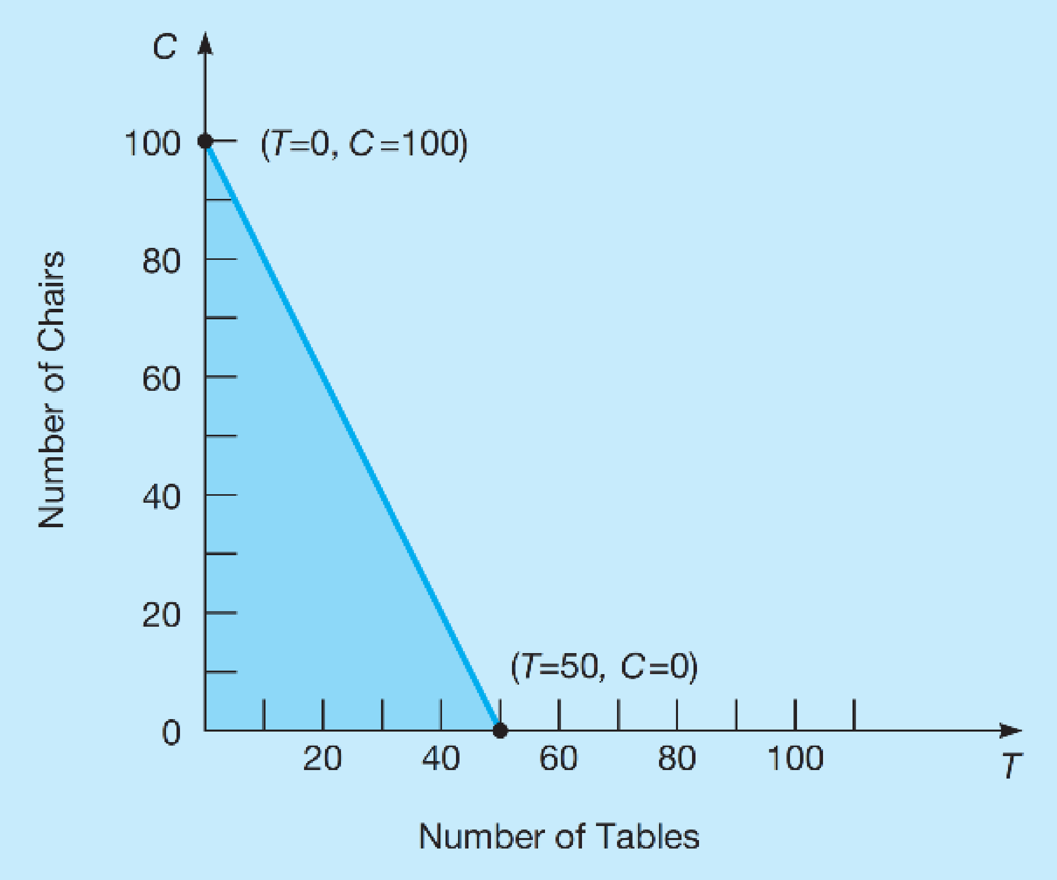
$$4(30) + 3(20) = 180$$

- The point (70, 40) lies above the line and does not satisfy the constraint

$$4(70) + 3(40) = 400$$

Graphical Representation of Constraints (7 of 11)

FIGURE 7.4 Region that Satisfies the Painting and Varnishing Constraint

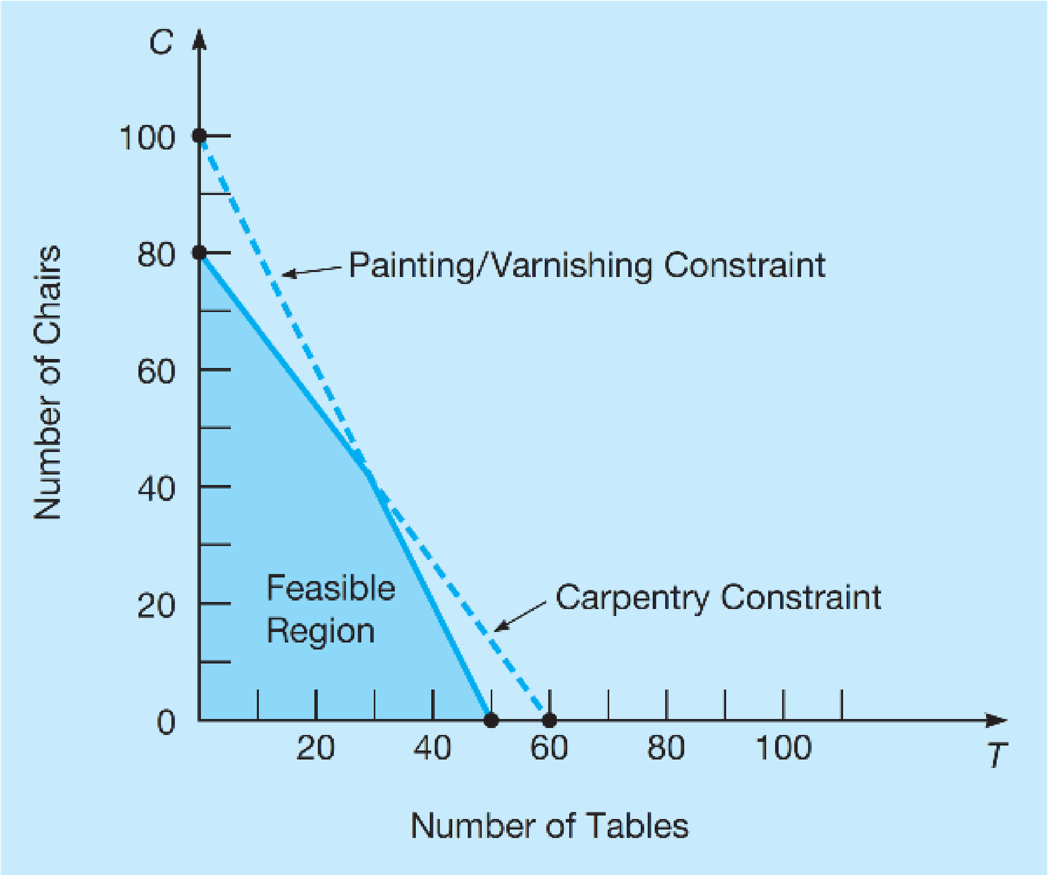


Graphical Representation of Constraints (8 of 11)

- To produce tables and chairs, both departments must be used
- Find a solution that satisfies both constraints *simultaneously*
- A new graph shows both constraint plots
- The **feasible region** is where all constraints are satisfied
  - Any point inside this region is a **feasible solution**
  - Any point outside the region is an **infeasible solution**

Graphical Representation of Constraints (9 of 11)

FIGURE 7.5 Feasible Solution Region for the Flair Furniture Company Problem



Graphical Representation of Constraints (10 of 11)

- For the point (30, 20)

Carpentry constraint	$4T + 3C \le 240$ hours available $(4)(30) + (3)(20) = 180$ hours used	✓
Painting constraint	$2T + 1C \le 100$ hours available $(2)(30) + (1)(20) = 80$ hours used	✓

- For the point (70, 40)

Carpentry constraint	$4T + 3C \le 240$ hours available $(4)(70) + (3)(40) = 400$ hours used	✓
Painting constraint	$2T + 1C \le 100$ hours available $(2)(70) + (1)(40) = 180$ hours used	✓

Graphical Representation of Constraints (11 of 11)

- For the point (50, 5)

Carpentry constraint	$4T + 3C \leq 240$ hours available $(4)(50) + (3)(5) = 215$ hours used	✔
Painting constraint	$2T + 1C \leq 100$ hours available $(2)(50) + (1)(5) = 105$ hours used	☐

Isoprofit Line Solution Method (1 of 7)

- Find the optimal solution from the many possible solutions
- Speediest method is to use the **isoprofit line**
- Starting with a small possible profit value, graph the objective function
- Move the objective function line in the direction of increasing profit while maintaining the slope
- The last point it touches in the feasible region is the optimal solution

Isoprofit Line Solution Method (2 of 7)

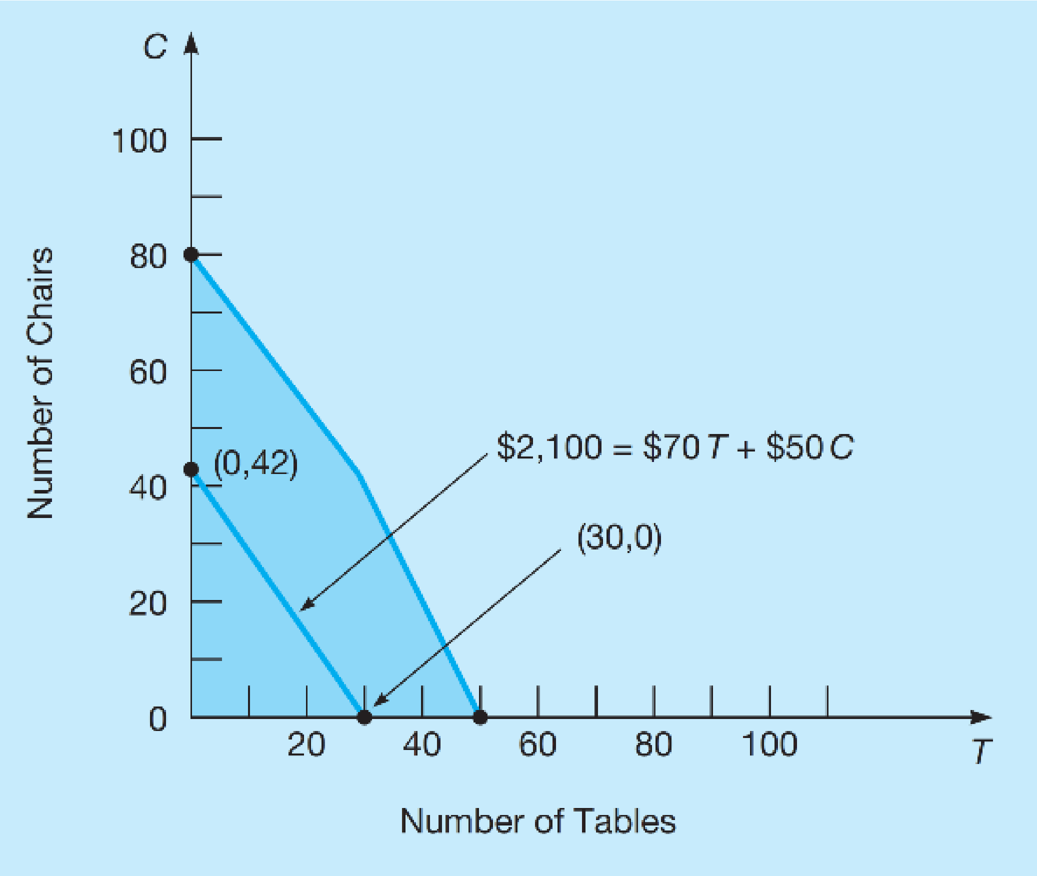
- Choose a profit of \$2,100
- The objective function is

$$\$2{,}100 = 70T + 50C$$

- Solving for the axis intercepts, draw the graph
- Obviously not the best possible solution
- Further graphs can be created using larger profits
  - The further we move from the origin, the larger the profit
- The highest profit (\$4,100) will be generated when the isoprofit line passes through the point (30, 40)

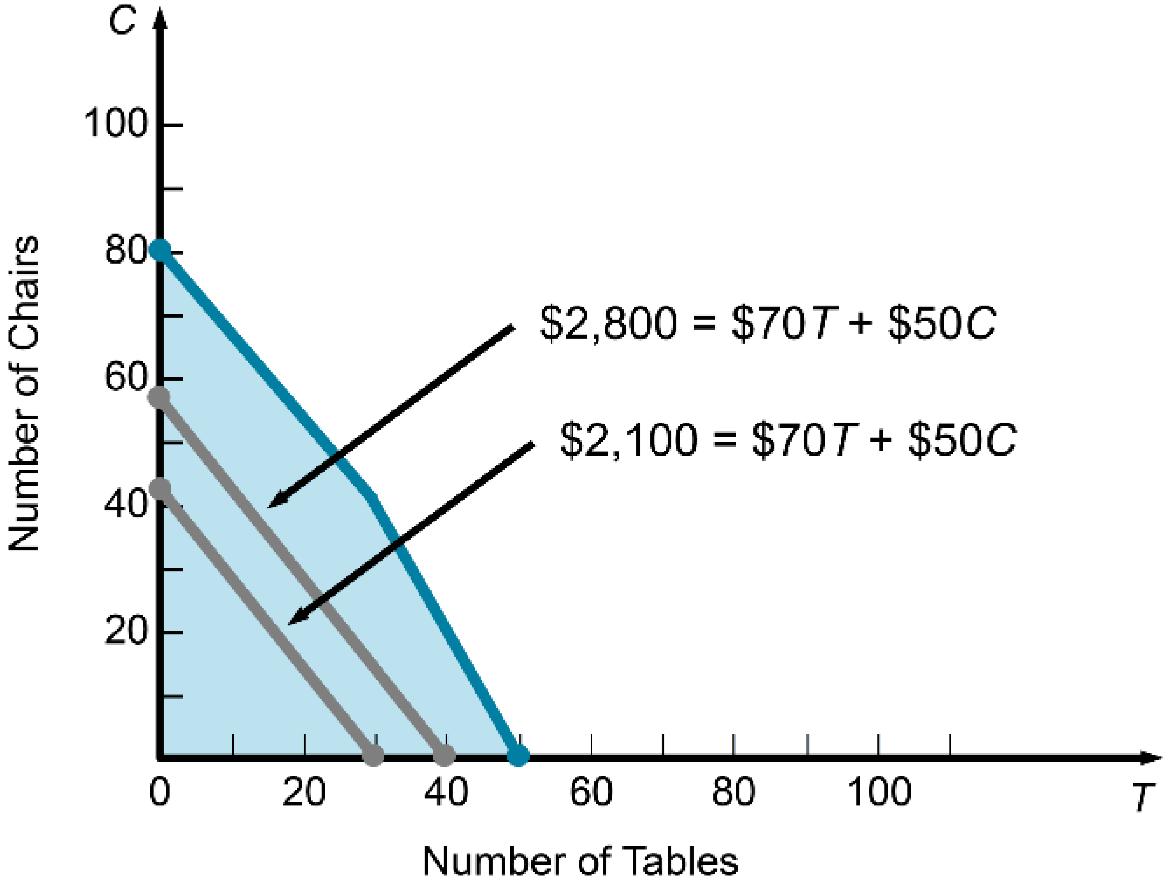
Isoprofit Line Solution Method (3 of 7)

FIGURE 7.6 Profit line of $2,100 Plotted for the Flair Furniture Company



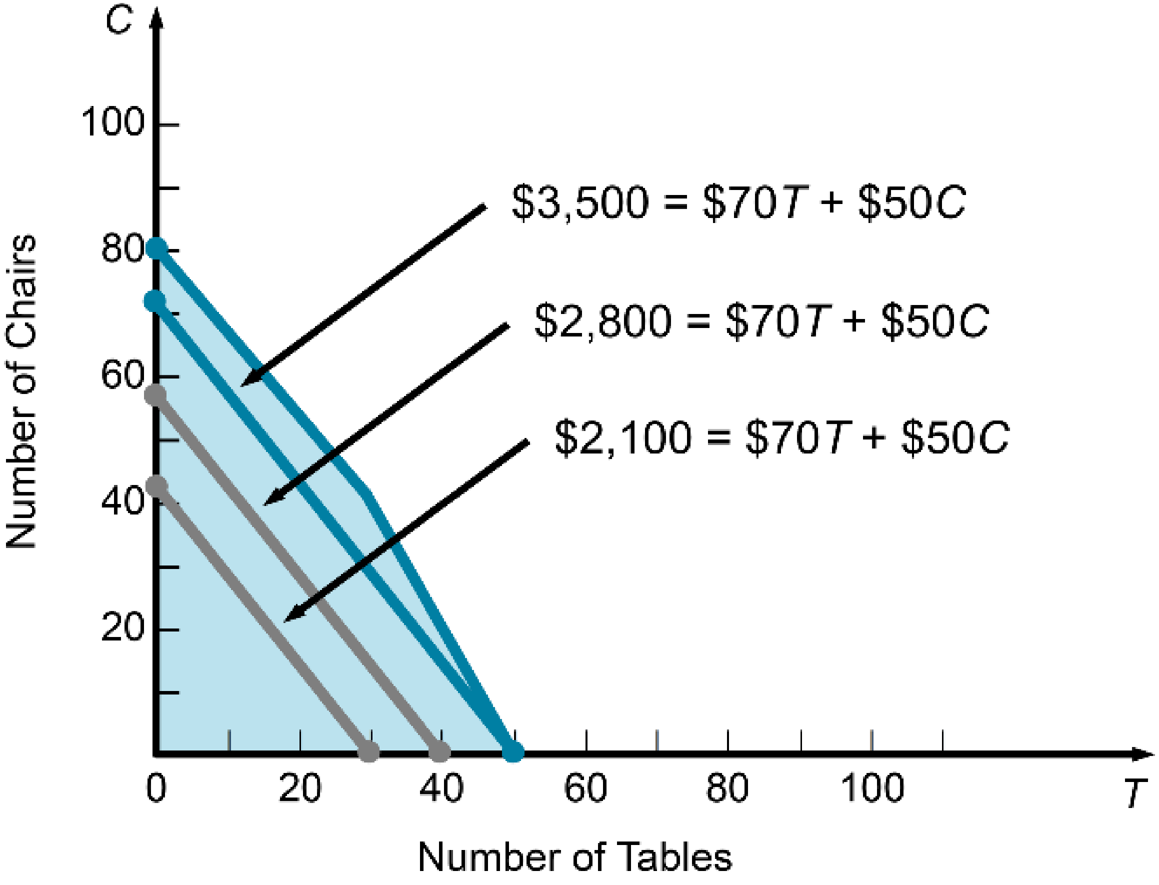
Isoprofit Line Solution Method (4 of 7)

FIGURE 7.7 Four Isoprofit Lines Plotted for the Flair Furniture Company



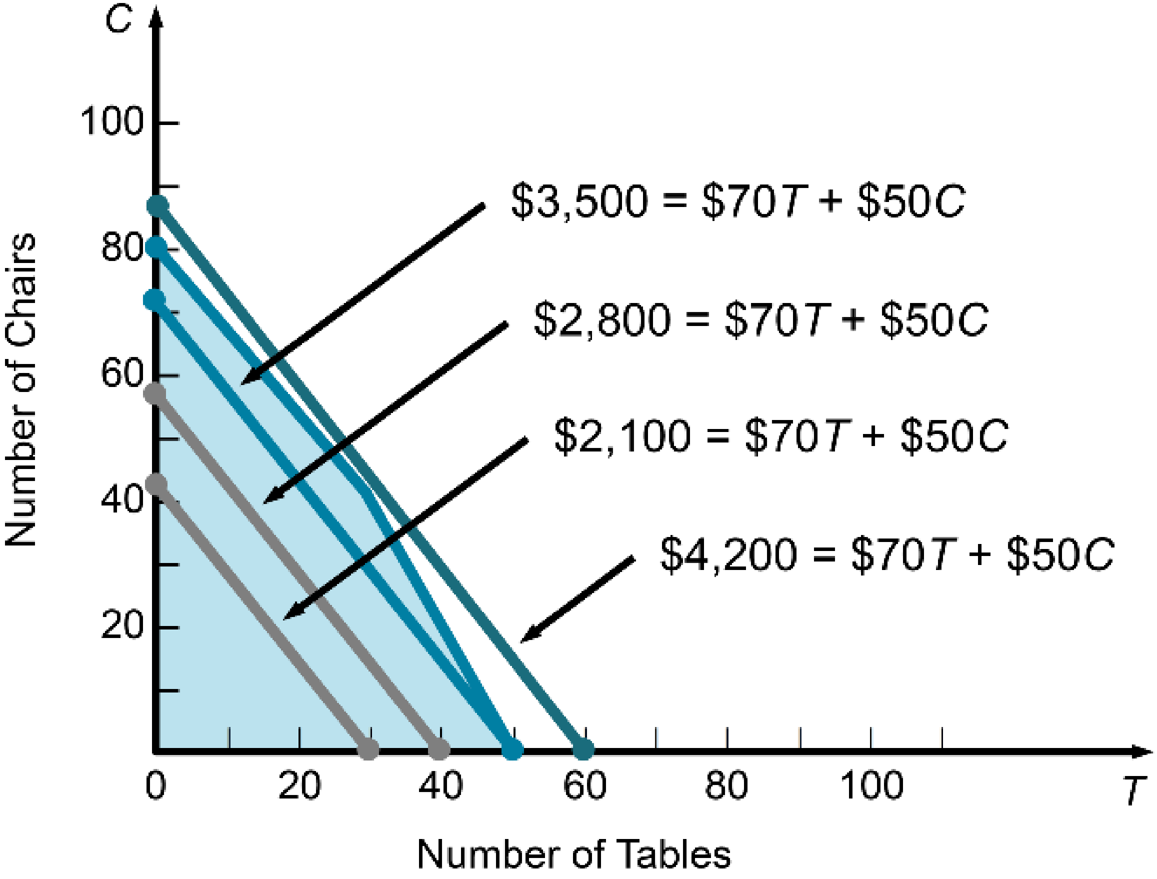
Isoprofit Line Solution Method (5 of 7)

FIGURE 7.7 Four Isoprofit Lines Plotted for the Flair Furniture Company



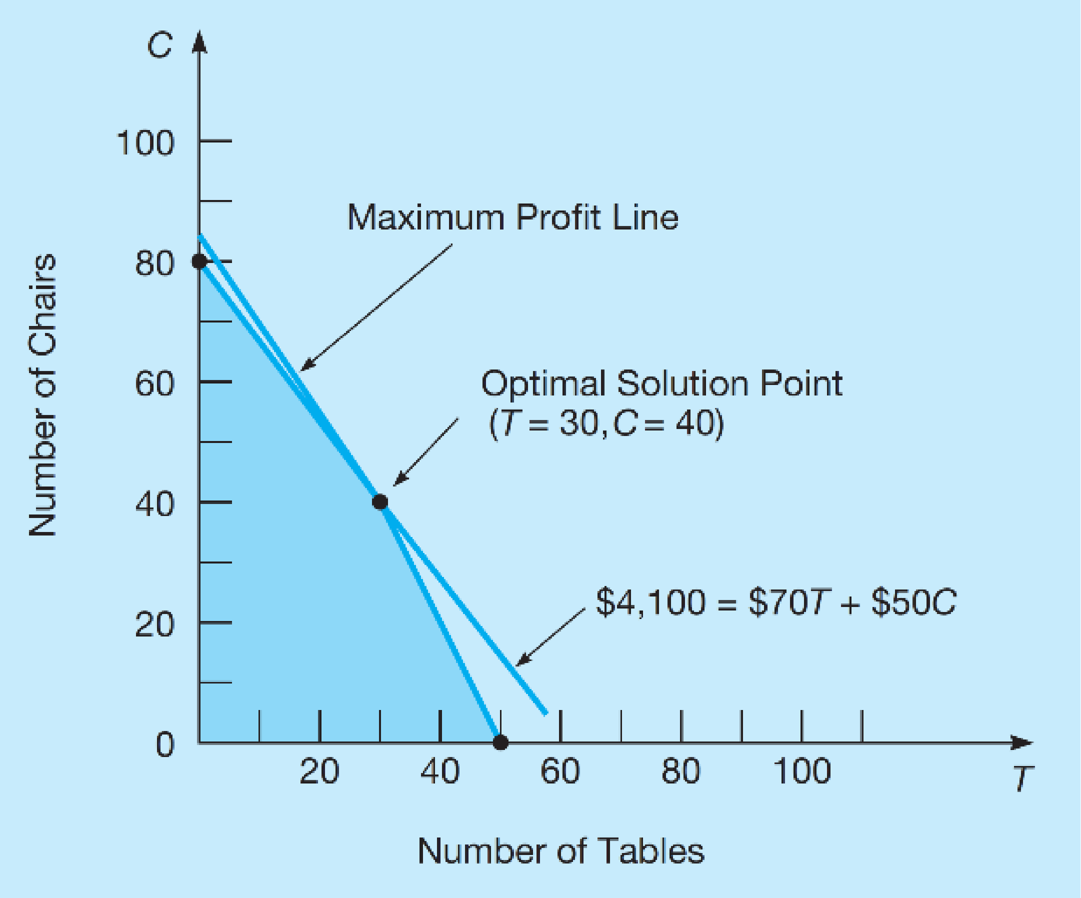
Isoprofit Line Solution Method (6 of 7)

FIGURE 7.7 Four Isoprofit Lines Plotted for the Flair Furniture Company



Isoprofit Line Solution Method (7 of 7)

FIGURE 7.8 Optimal Solution to the Flair Furniture Problem

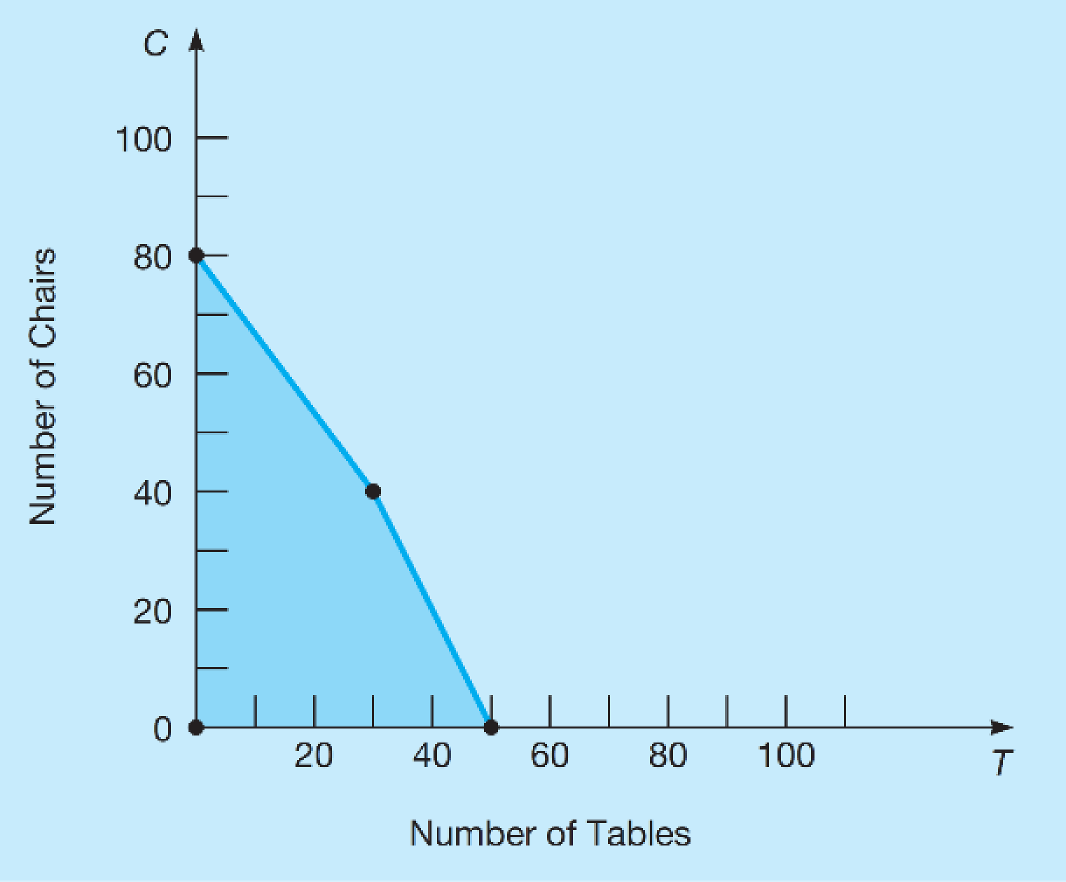


Corner Point Solution Method (1 of 4)

- The **corner point method** for solving LP problems
- Look at the profit at every corner point of the feasible region
- Mathematical theory is that an optimal solution must lie at one of the **corner points** or **extreme points**

Corner Point Solution Method (2 of 4)

FIGURE 7.9 Four Corner Points of the Feasible Region



Corner Point Solution Method (3 of 4)

- Solve for the intersection of the two constraint lines
- Using the elimination method to solve simultaneous equations method, select a variable to be eliminated
- Eliminate T by multiplying the second equation by −2 and add it to the first equation

$$-2(2T + 1C = 100) = -4T - 2C = -200$$

$$\begin{array}{rl} 4T + 3C = \ \ 240 & \text{(carpentry)} \\ \underline{-4T - 2C = -200} & \text{(painting)} \\ C = \ \ \ 40 & \end{array}$$

Corner Point Solution Method (4 of 4)

- Substitute $C = 40$ into either equation to solve for T

$$4T + 3(40) = 240$$
$$4T + 120 = 240$$
$$4T = 120$$
$$T = 30$$

Thus the corner point is (30, 40)

Highest profit – Optimal Solution

TABLE 7.3 Feasible Corner Points and Profits for Flair Furniture

NUMBER OF TABLES (T)	NUMBER OF CHAIRS (C)	PROFIT = $70T + $50C
0	0	$0
50	0	$3,500
0	80	$4,000
30	40	$4,100

Slack and Surplus (1 of 3)

- **Slack** is the amount of a resource that is not used
  - For a less-than-or-equal constraint

Slack = (Amount of resource available)
− (Amount of resource used)

  - Flair decides to produce 20 tables and 25 chairs

4(20) + 3(25) = 155 (carpentry time used)

240 = (carpentry time available)

240 − 155 = 85 (Slack time in carpentry)

Slack and Surplus (2

At the optimal solution, slack is 0 as all 240 hours are used

- **Slack** is the amount of a resou
  - For a less-than-or-equal constraint

Slack = (Amount of resource available)
− (Amount of resource used)

  - Flair decides to produce 20 tables and 25 chairs

4(20) + 3(25) = 155 (carpentry time used)

240 = (carpentry time available)

240 − 155 = 85 (Slack time in carpentry)

Slack and Surplus (3 of 3)

- **Surplus** is used with a greater-than-or-equal-to constraint to indicate the amount by which the right-hand side of the constraint is exceeded

$$\text{Surplus} = (\text{Actual amount}) - (\text{Minimum amount})$$

  - New constraint

$$T + C \geq 42$$

  - If $T = 20$ and $C = 25$, then

$$20 + 25 = 45$$

$$\text{Surplus} = 45 - 42 = 3$$

Summaries of Graphical Solution Methods

TABLE 7.4 Summaries of Graphical Solution Methods

ISOPROFIT METHOD

1. Graph all constraints and find the feasible region.
2. Select a specific profit (or cost) line and graph it to find the slope.
3. Move the objective function line in the direction of increasing profit (or decreasing cost) while maintaining the slope. The last point it touches in the feasible region is the optimal solution.
4. Find the values of the decision variables at this last point and compute the profit (or cost).

CORNER POINT METHOD

1. Graph all constraints and find the feasible region.
2. Find the corner points of the feasible region.
3. Compute the profit (or cost) at each of the feasible corner points.
4. Select the corner point with the best value of the objective function found in Step 3. This is the optimal solution.

Solving Flair Furniture's LP Problem

- Most organizations have access to software to solve big LP problems
- There are differences between software implementations, the approach is basically the same
- With experience with computerized LP algorithms, it is easy to adjust to minor changes

Solving Minimization Problems

- Many LP problems involve minimizing an objective such as cost
- Minimization problems can be solved graphically
  - Set up the feasible solution region
  - Use either the corner point method or an isocost line approach
  - Find the values of the decision variables (e.g., X_1 and X_2) that yield the minimum cost

Holiday Meal Turkey Ranch (1 of 10)

- The Holiday Meal Turkey Ranch is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys

TABLE 7.5 Holiday Meal Turkey Ranch data

	COMPOSITION OF EACH POUND OF FEED (OZ.)		
INGREDIENT	**BRAND 1 FEED**	**BRAND 2 FEED**	**MINIMUM MONTHLY REQUIREMENT PER TURKEY (OZ.)**
A	5	10	90
B	4	3	48
C	0.5	0	1.5
Cost per pound	2 cents	3 cents	

Holiday Meal Turkey Ranch (2 of 10)

Let

X_1 = number of pounds of brand 1 feed purchased

X_2 = number of pounds of brand 2 feed purchased

Minimize cost (in cents) = $2X_1 + 3X_2$

subject to:

$5X_1 + 10X_2 \geq 90$ ounces (ingredient A constraint)

$4X_1 + 3X_2 \geq 48$ ounces (ingredient B constraint)

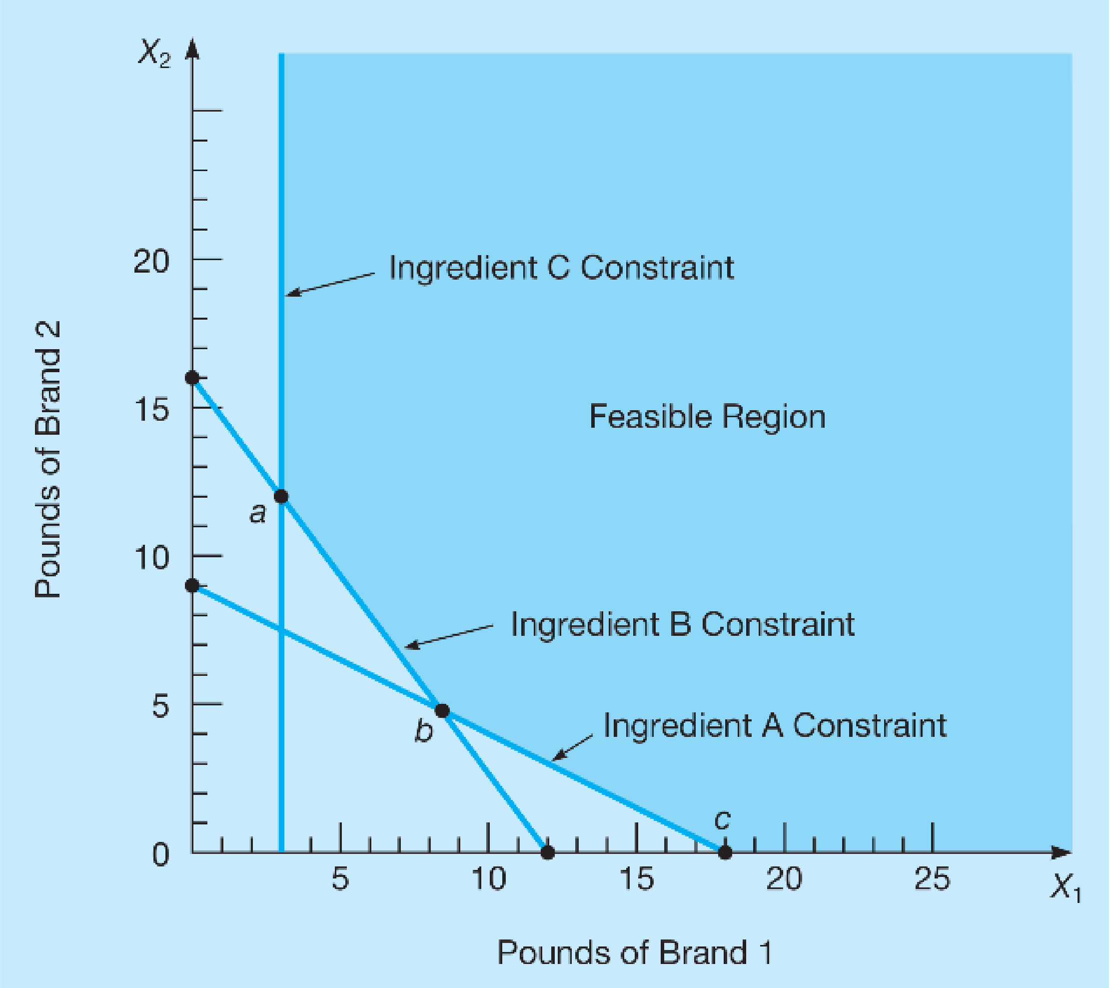
$0.5X_1 \geq 1.5$ ounces (ingredient C constraint)

$X_1 \geq 0$ (nonnegativity constraint)

$X_2 \geq 0$ (nonnegativity constraint)

Holiday Meal Turkey Ranch (3 of 10)

FIGURE 7.10
Feasible Region for the Holiday Meal Turkey Ranch Problem



Holiday Meal Turkey Ranch (4 of 10)

- Solve for the values of the three corner points
  - Point a is the intersection of ingredient constraints C and B

$$4X_1 + 3X_2 = 48$$

$$X_1 = 3$$

  - Substituting 3 in the first equation, we find $X_2 = 12$
  - Solving for point b we find $X_1 = 8.4$ and $X_2 = 4.8$
  - Solving for point c we find $X_1 = 18$ and $X_2 = 0$

Holiday Meal Turkey Ranch (5 of 10)

- Substituting these values back into the objective function we find

$$\text{Cost} = 2X_1 + 3X_2$$

$$\text{Cost at point } a = 2(3) + 3(12) = 42$$

$$\boxed{\text{Cost at point } b = 2(8.4) + 3(4.8) = 31.2}$$

$$\text{Cost at point } c = 2(18) + 3(0) = 36$$

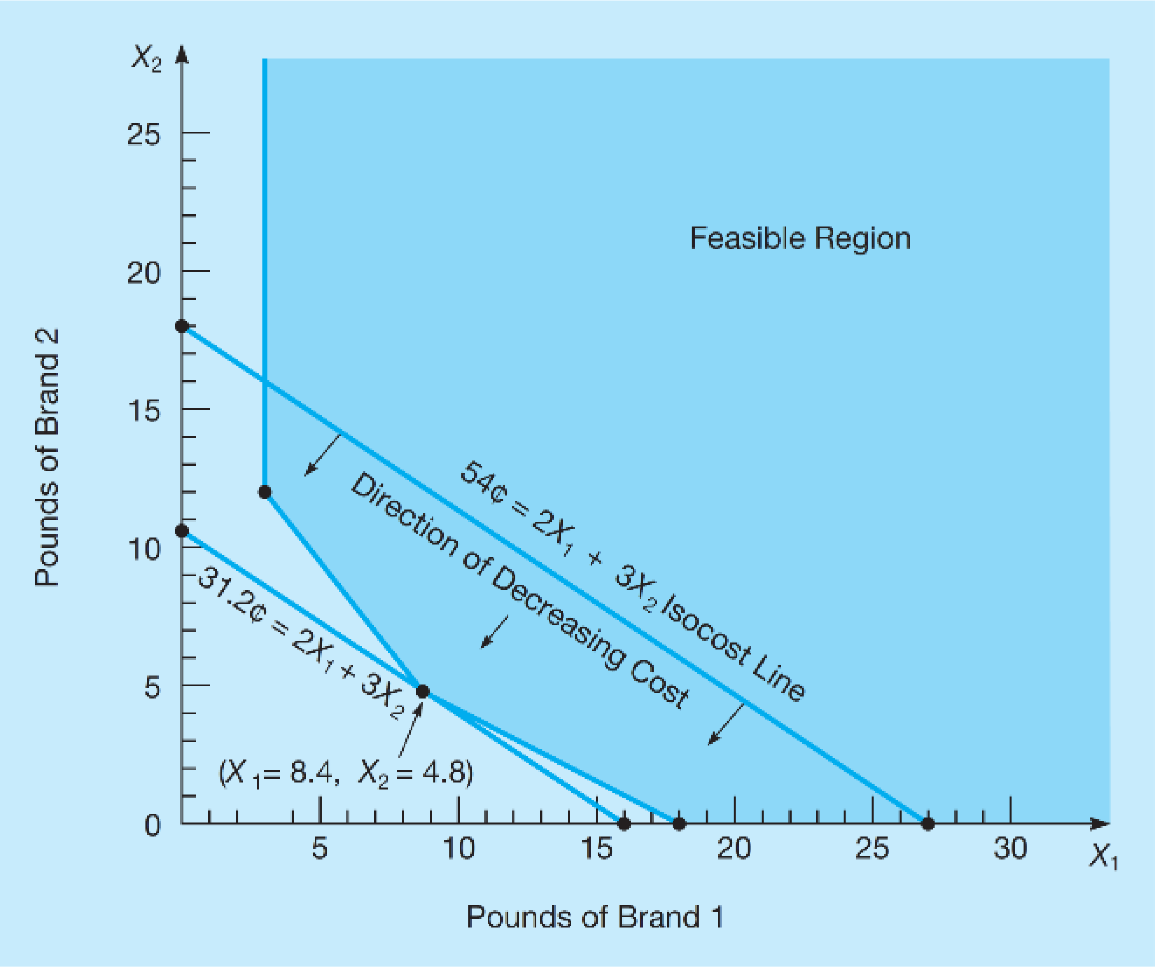
  - The lowest cost solution is to purchase 8.4 pounds of brand 1 feed and 4.8 pounds of brand 2 feed for a total cost of 31.2 cents per turkey

Holiday Meal Turkey Ranch (6 of 10)

- Solving using an **isocost line**
- Move the isocost line toward the lower left
- The last point touched in the feasible region will be the optimal solution

Holiday Meal Turkey Ranch (7 of 10)

FIGURE 7.11
Graphical Solution to the Holiday Meal Turkey Ranch Problem Using the Isocost Line



Four Special Cases in LP

- Four special cases and difficulties arise at times when using the graphical approach
  1. No feasible solution
  2. Unboundedness
  3. Redundancy
  4. Alternate Optimal Solutions

No Feasible Solution (1 of 2)

- No solution to the problem that satisfies all the constraint equations
- No feasible solution region exists
- A common occurrence in the real world
- Generally one or more constraints are relaxed until a solution is found
- Consider the following three constraints

$$X_1 + 2X_2 \leq 6$$

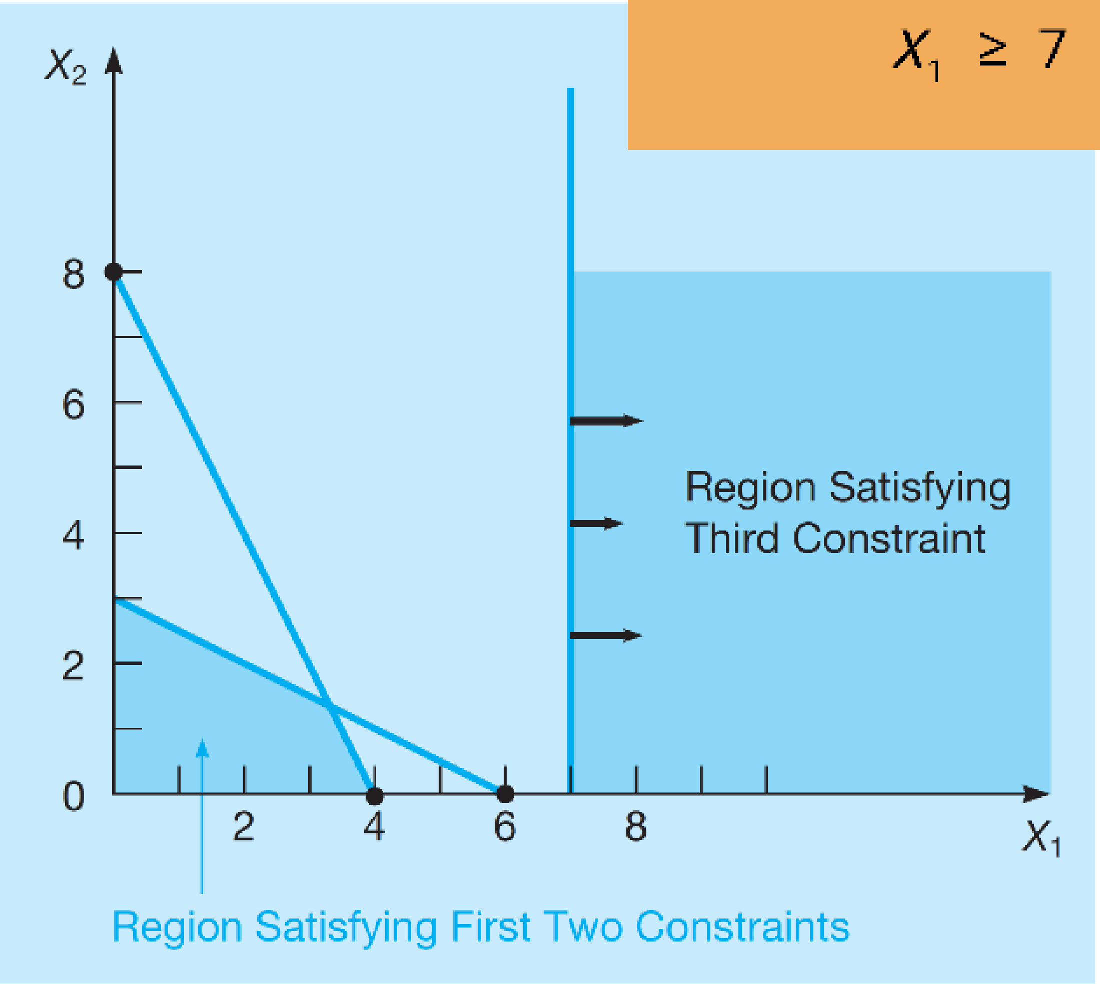
$$2X_1 + X_2 \leq 8$$

$$X_1 \geq 7$$

No Feasible Solution (2 of 2)

$$X_1 + 2X_2 \le 6$$
$$2X_1 + X_2 \le 8$$
$$X_1 \ge 7$$

FIGURE 7.12 A problem with no feasible solution

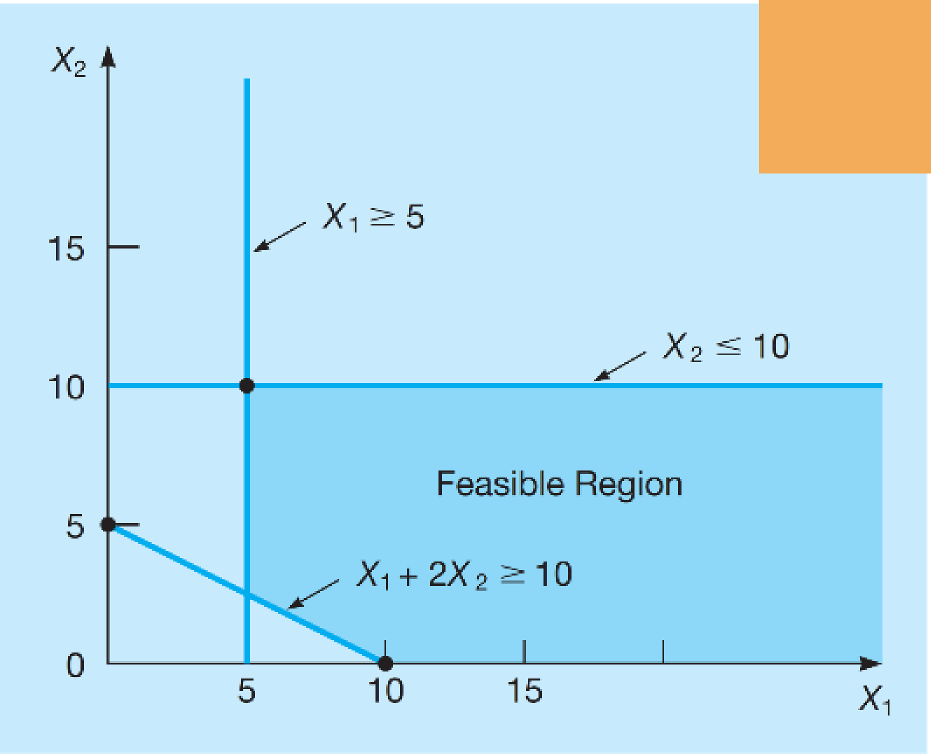


Unboundedness (1 of 2)

- Sometimes a linear program will not have a finite solution
- In a maximization problem
  - One or more solution variables, and the profit, can be made infinitely large without violating any constraints
- In a graphical solution, the feasible region will be open ended
- Usually means the problem has been formulated improperly

Unboundedness (2 of 2)

FIGURE 7.13 A Feasible Region That Is Unbounded to the Right



$$
\begin{aligned}
\text{Maximize profit} = \ & \$3X_1 + \$5X_2 \\
\text{subject to} \quad & X_1 \geq 5 \\
& X_2 \leq 10 \\
& X_1 + 2X_2 \geq 10 \\
& X_1, X_2 \geq 0
\end{aligned}
$$

Redundancy (1 of 2)

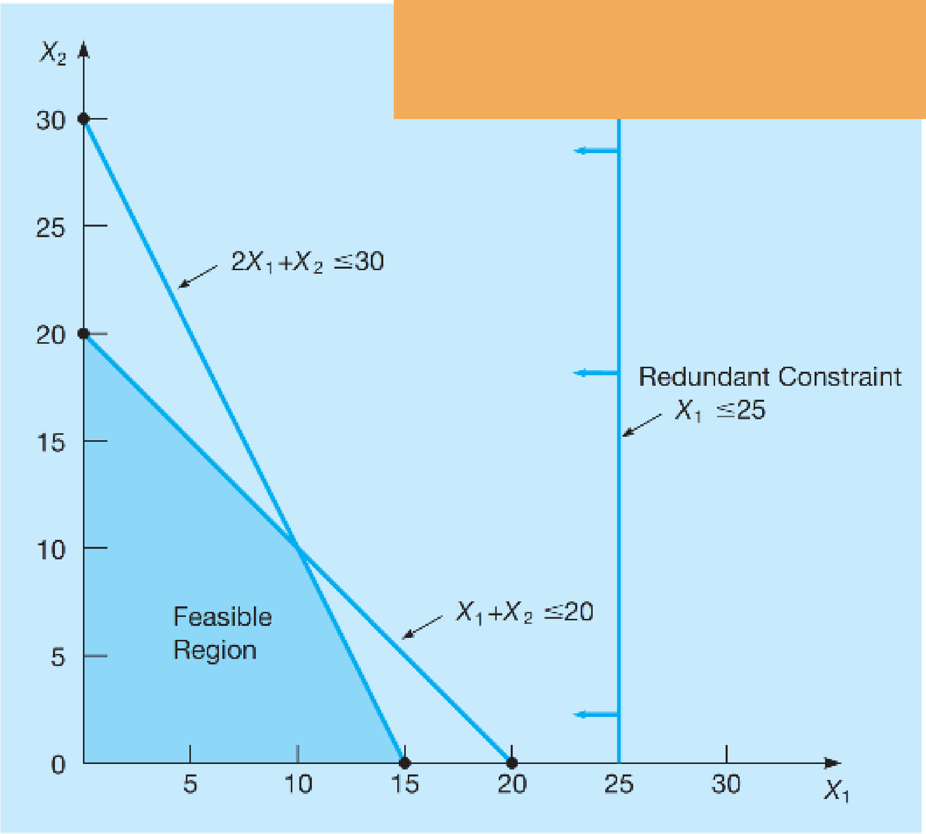
- A redundant constraint is one that does not affect the feasible solution region
- One or more constraints may be binding
- This is a very common occurrence in the real world
- Causes no particular problems, but eliminating redundant constraints simplifies the model

$$
\begin{array}{llll}
\text{Maximize profit} = & \$1X_1 & + \$2X_2 & \\
\text{subject to} & X_1 & + X_2 & \le 20 \\
 & 2X_1 & + X_2 & \le 30 \\
 & X_1 & & \le 25 \\
 & & X_1, X_2 & \ge 0
\end{array}
$$

Redundancy (2 of 2)

$$\begin{array}{llllll}
\text{Maximize profit} = & \$1X_1 & + \$2X_2 & \\
\text{subject to} & X_1 & + X_2 & \leq 20 \\
 & 2X_1 & + X_2 & \leq 30 \\
 & X_1 & & \leq 25 \\
 & & X_1, X_2 & \geq 0
\end{array}$$

FIGURE 7.14
Problem with a
Redundant C



Alternate Optimal Solutions (1 of 2)

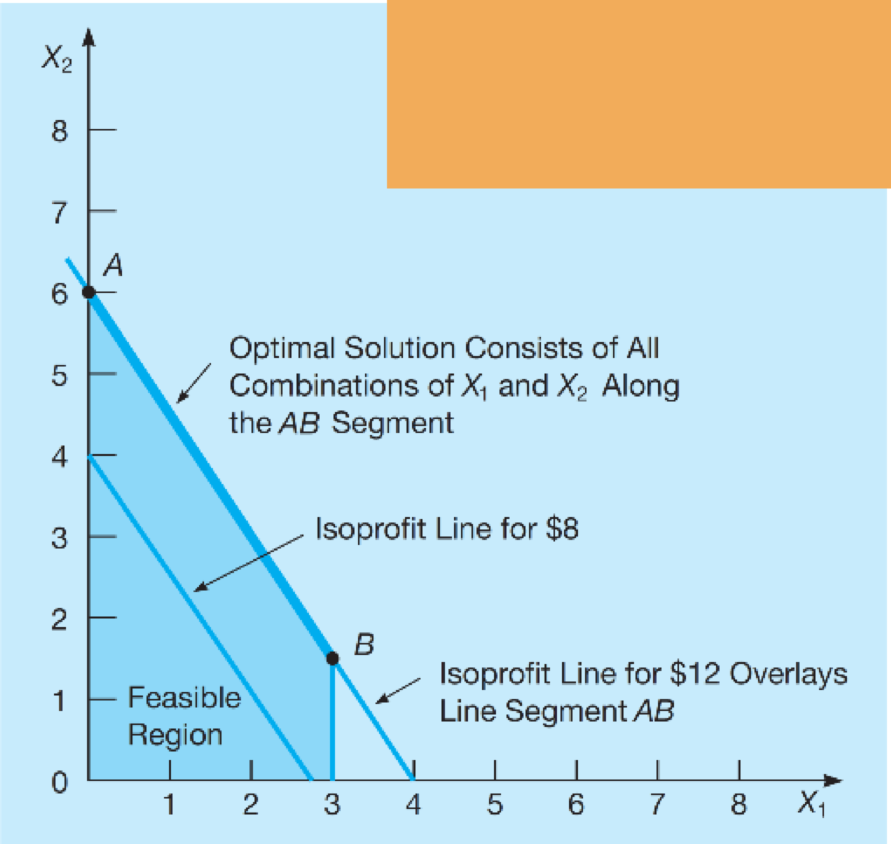
- Occasionally two or more optimal solutions may exist
- Graphically this occurs when the objective function's isoprofit or isocost line runs perfectly parallel to one of the constraints
- Allows management great flexibility in deciding which combination to select as the profit is the same at each alternate solution

$$\begin{array}{llll}
\text{Maximize profit} = & \$3X_1 & + \$2X_2 & \\
\text{subject to} & 6X_1 & + 4X_2 & \leq 24 \\
 & X_1 & & \leq 3 \\
 & & X_1, X_2 & \geq 0
\end{array}$$

Alternate Optimal Solutions (2 of 2)

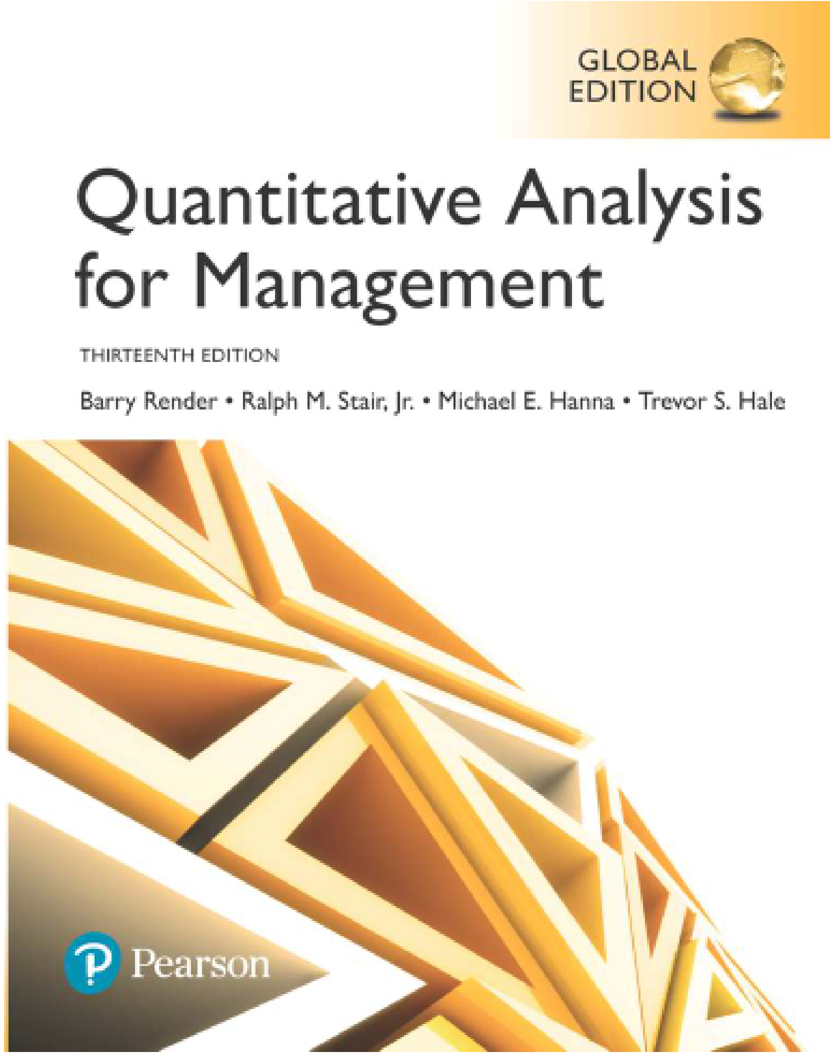
FIGURE 7.15 Example of Alternate Optimal Solutions

Maximize profit =	$\$3X_1$	$+ \$2X_2$	
subject to	$6X_1$	$+ 4X_2$	≤ 24
	X_1		≤ 3
		X_1, X_2	≥ 0



Quantitative Analysis for Management

Thirteenth Edition, Global Edition



Module 7

Linear Programming: The Simplex Method

Pearson

Introduction

- Most real-life LP problems have more than two variables and cannot be solved using the graphical procedure
- The **simplex method**
- Examines the corner points in a systematic fashion
    - An **iterative** process
    - Each iteration improves the value of the objective function
    - Yields optimal solution and other valuable economic information

How to Set Up the Initial Simplex Solution

- Flair Furniture Company problem

T = number of tables produced

C = number of chairs produced

Maximize profit = $\$70T + \$50C$ (objective function)

subject to $2T + 1C \leq 100$ (painting hours constraint)

$4T + 3C \leq 240$ (carpentry hours constraint)

$T, C \geq 0$ (nonnegativity constraints)

Converting the Constraints to Equations (1 of 3)

- Convert inequality constraints into an equation
- Add a **slack variable** to less-than-or-equal-to constraints

S_1 = slack variable representing unused hours in the painting department

S_2 = slack variable representing unused hours in the carpentry department

$$2T + 1C + S_1 = 100$$
$$4T + 3C + S_2 = 240$$

Converting the Constraints to Equations (2 of 3)

- If Flair produces $T = 40$ and $C = 10$

$$\begin{aligned} 2T + 1C + S_1 &= 100 \\ 2(40) + 1(10) + S_1 &= 100 \\ S_1 &= 10 \end{aligned}$$

- Adding all variables into all equations,

$$\begin{aligned} 2T + 1C + 1S_1 + 0S_2 &= 100 \\ 4T + 3C + 0S_1 + 1S_2 &= 240 \\ T, C, S_1, S_2 &\geq 0 \end{aligned}$$

Converting the Constraints to Equations (3 of 3)

- If Flair produces $T = 40$ and $C = 10$

Objective function becomes

Maximize profit = $\$70T + \$50C + \$0S_1 + \$0S_2$

- Adding all variables into all equations,

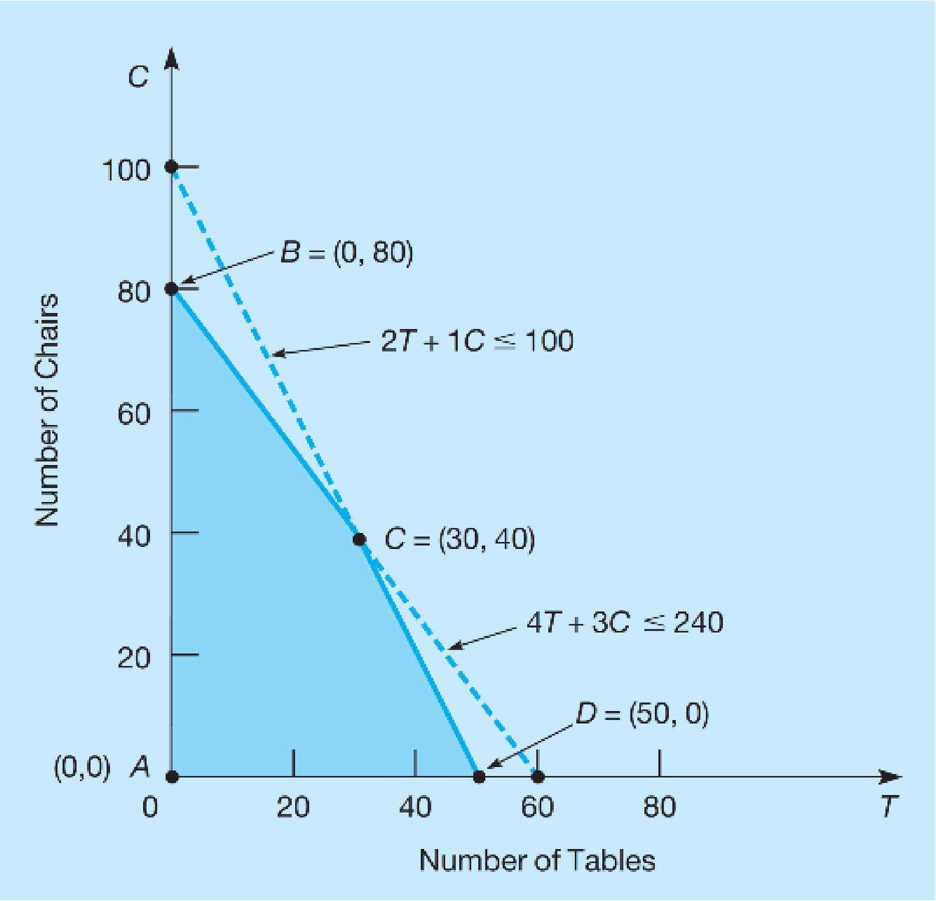
$$2T + 1C + 1S_1 + 0S_2 = 100$$

$$4T + 3C + 0S_1 + 1S_2 = 240$$

$$T, C, S_1, S_2 \geq 0$$

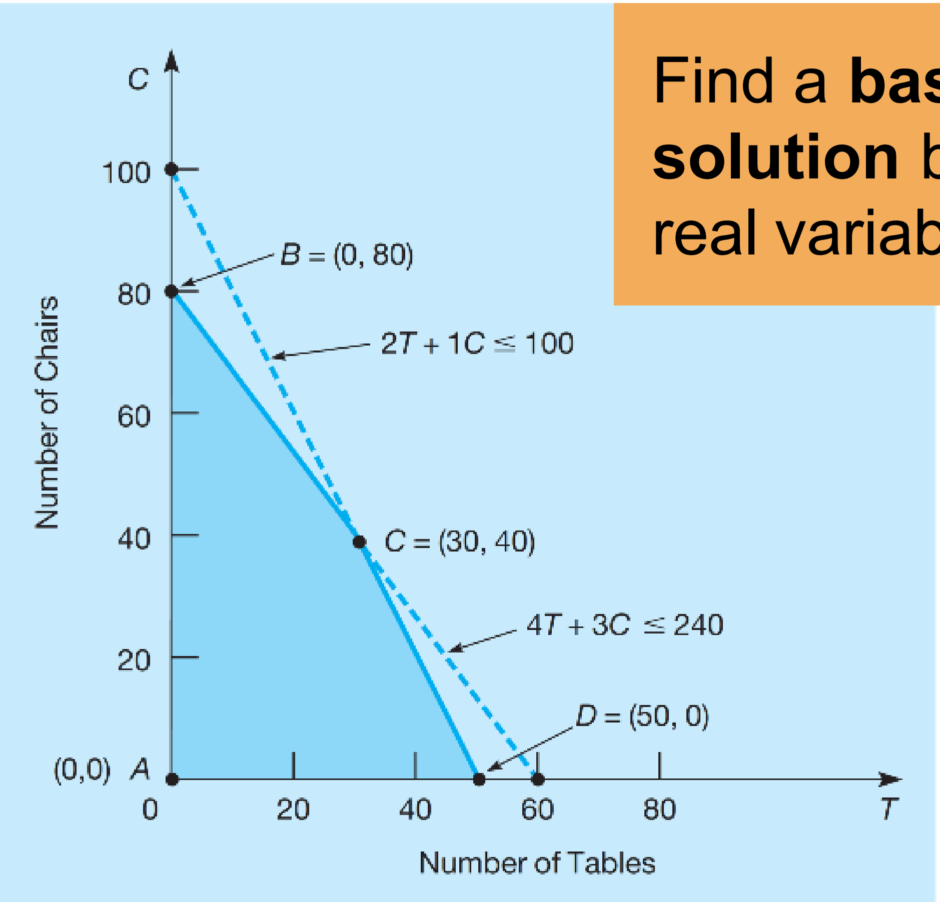
Finding an Initial Solution Algebraically (1 of 2)

FIGURE M7.1 Corner Points of the Flair Furniture Company Problem



Finding an Initial Solution Algebraically (2 of 2)

FIGURE M7.1 Corner Points of the Flair Furniture Company Problem



Find a **basic feasible solution** by setting all real variables to 0

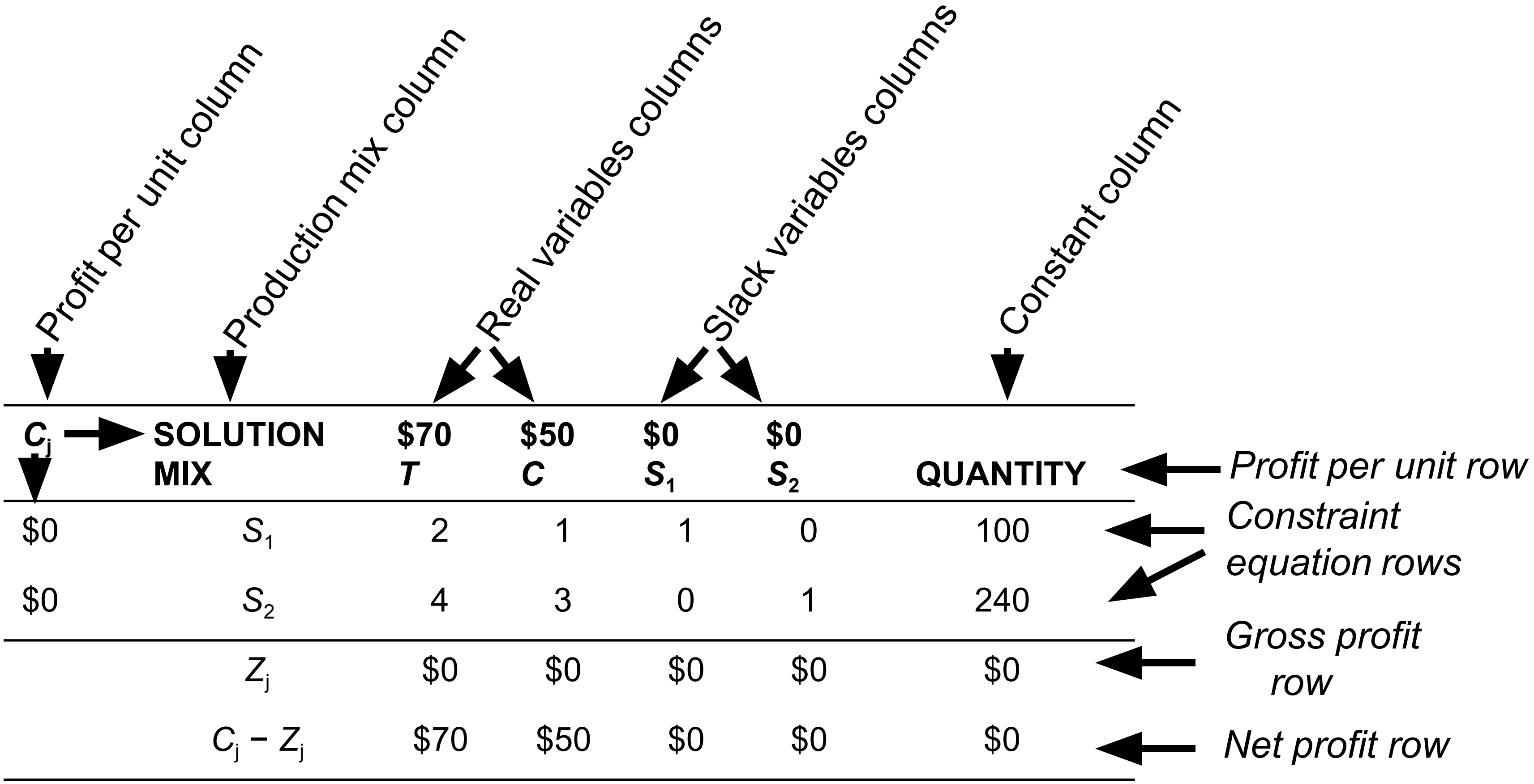
The First Simplex Tableau (1 of 10)

- Constraint equations
  - Place all of the coefficients into tabular form, the first **simplex tableau**

SOLUTION MIX	T	C	S_1	S_2	QUANTITY (RIGHT-HAND SIDE [RHS])
S_1	2	1	1	0	100
S_2	4	3	0	1	240

The First Simplex Tableau (2 of 10)

TABLE M7.1 Flair Furniture's Initial Simplex Tableau



Profit per unit column

Production mix column

Real variables columns

Slack variables columns

Constant column

C_j	SOLUTION	$70	$50	$0	$0	
	MIX	T	C	S_1	S_2	QUANTITY
$0	S_1	2	1	1	0	100
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$70	$50	$0	$0	$0

Profit per unit row

Constraint equation rows

Gross profit row

Net profit row

The First Simplex Tableau (3 of 10)

- Begin the initial solution procedure at the origin
- Slack variables are nonzero and are the *initial* **solution mix**
- Values in the *quantity column*
- Initial solution is the *basic feasible solution*

$$\begin{bmatrix} T \\ C \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 240 \end{bmatrix}$$

- Variables in the **solution mix**, the **basis**, are called **basic variables**
- Those not in the basis are called **nonbasic variables**

The First Simplex Tableau (4 of 10)

- Optimal solution in vector form
- T and C are the final basic variables
- S_1 and S_2 are nonbasic variables

$$\begin{bmatrix} T \\ C \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \\ 0 \\ 0 \end{bmatrix}$$

The First Simplex Tableau (5 of 10)

- Substitution rates

$$\text{Under } T \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \text{Under } C \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \text{Under } S_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{Under } S_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

  – For every unit of T introduced into the **current solution**, 2 units of S_1 and 4 units of S_2 must be removed

For any variable ever to appear in the solution mix column, it must have the number 1 someplace in its column and 0s in every other place in that column

The First Simplex Tableau (6 of 10)

For three less-than-or-equal-to constraints

SOLUTION MIX	S_1	S_2	S_3
S_1	1	0	0
S_2	0	1	0
S_3	0	0	1

For any variable ever to appear in the solution mix column, it must have the number 1 someplace in its column and 0s in every other place

The First Simplex Tableau (7 of 10)

- Adding the Objective Function

C_j →		**\$70**	**\$50**	**\$0**	**\$0**	
↓	**SOLUTION MIX**	T	C	S_1	S_2	**QUANTITY**
\$0	S_1	2	1	1	0	100
\$0	S_2	4	3	0	1	240

The First Simplex Tableau (8 of 10)

- The Z_j and $C_j - Z_j$ Rows

C_j	SOLUTION MIX	\$70 T	\$50 C	\$0 S_1	\$0 S_2	QUANTITY
\$0	S_1	2	1	1	0	100
\$0	S_2	4	3	0	1	240
	Z_j	\$0	\$0	\$0	\$0	\$0
	$C_j - Z_j$	\$70	\$50	\$0	\$0	\$0

  - Compute the Z_j value for each column of the initial solution by multiplying the 0 contribution value of each number in the C_j column by each number in that row and the *j*th column and summing

Z_j (for gross profit) = (Profit per unit of S_1) × (Number of units of S_1)

+ (Profit per unit of S_2) × (Number of units of S_2)

= \$0 × 100 units + \$0 × 240 units

= \$0 profit

The First Simplex Tableau (9 of 10)

- The Z_j and $C_j - Z_j$ Rows

C_j	SOLUTION MIX	\$70 T	\$50 C	\$0 S_1	\$0 S_2	QUANTITY
\$0	S_1	2	1	1	0	100
\$0	S_2	4	3	0	1	240
	Z_j	\$0	\$0	\$0	\$0	\$0
	$C_j - Z_j$	\$70	\$50	\$0	\$0	\$0

Z_j (for column T) = (\$0)(2) + (\$0)(4) = \$0

Z_j (for column C) = (\$0)(1) + (\$0)(3) = \$0

Z_j (for column S_1) = (\$0)(1) + (\$0)(0) = \$0

Z_j (for column S_2) = (\$0)(0) + (\$0)(1) = \$0

The First Simplex Tableau (10 of 10)

- The Z_j and $C_j - Z_j$ Rows

C_j	**SOLUTION**	**$70**	**$50**	**$0**	**$0**	
	MIX	***T***	***C***	S_1	S_2	**QUANTITY**
$0	S_1	2	1	1	0	100
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$70	$50	$0	$0	$0

	COLUMN			
	T	***C***	S_1	S_2
C_j for column	$70	$50	$0	$0
Z_j for column	0	0	0	0
$C_j - Z_j$ for column	$70	$50	$0	$0

Simplex Solution Procedures (1 of 2)

1. Determine which variable to enter into the solution mix next. One way of doing this is by identifying the column, and hence the variable, with the largest positive number in the $C_j - Z_j$ row of the preceding tableau. The column identified in this step is called the **pivot column**.

2. Determine which variable to replace. Decide which basic variable currently in the solution will have to leave to make room for the new variable. Divide each amount in the **quantity column** by the corresponding number in the column selected in step 1. The row with the *smallest nonnegative number* calculated in this fashion will be replaced in the next tableau. This row is often referred to as the **pivot row**. The number at the intersection of the pivot row and pivot column is referred to as the **pivot number**.

Simplex Solution Procedures (2 of 2)

3. Compute new values for the pivot row by dividing every number in the row by the pivot number.
4. Compute the new values for each remaining row. All remaining row(s) are calculated as follows:

$$\left(\text{New row numbers}\right) = \left(\text{Numbers in old row}\right) - \left[\begin{pmatrix}\text{Number above}\\ \text{or below}\\ \text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{Corresponding number in}\\ \text{the new row, that is, the}\\ \text{row replaced in step 3}\end{pmatrix}\right]$$

5. Compute the Z_j and $C_j - Z_j$ rows, as demonstrated in the initial tableau. If all numbers in the $C_j - Z_j$ row are 0 or negative, an optimal solution has been reached. If this is not the case, return to step 1.

The Second Simplex Tableau (1 of 10)

- Step 1 – Select variable to enter, *T*

TABLE M7.2 Pivot Column Identified in the Initial Simplex Tableau

C_j → ↓	SOLUTION MIX	\$70 T	\$50 C	\$0 S_1	\$0 S_2	QUANTITY (RHS)
\$0	S_1	2	1	1	0	100
\$0	S_2	4	3	0	1	240
	Z_j	\$0	\$0	\$0	\$0	\$0
	$C_j - Z_j$	\$70	\$50	\$0	\$0	

Pivot column

Total profit

The Second Simplex Tableau (2 of 10)

- Step 2 – Select the variable to be replaced, S_1

For S_1,

$$\frac{100\left(\text{hours of painting time available}\right)}{2\left(\text{hours required per table}\right)} = 50 \text{ tables}$$

For S_2,

$$\frac{240\left(\text{hours of painting time available}\right)}{4\left(\text{hours required per table}\right)} = 60 \text{ tables}$$

The Second Simplex Tableau (3 of 10)

- Step 2 – Select the variable to be replaced, S_1

TABLE M7.3 Pivot Row and Pivot Number Identified in the Initial Simplex Tableau

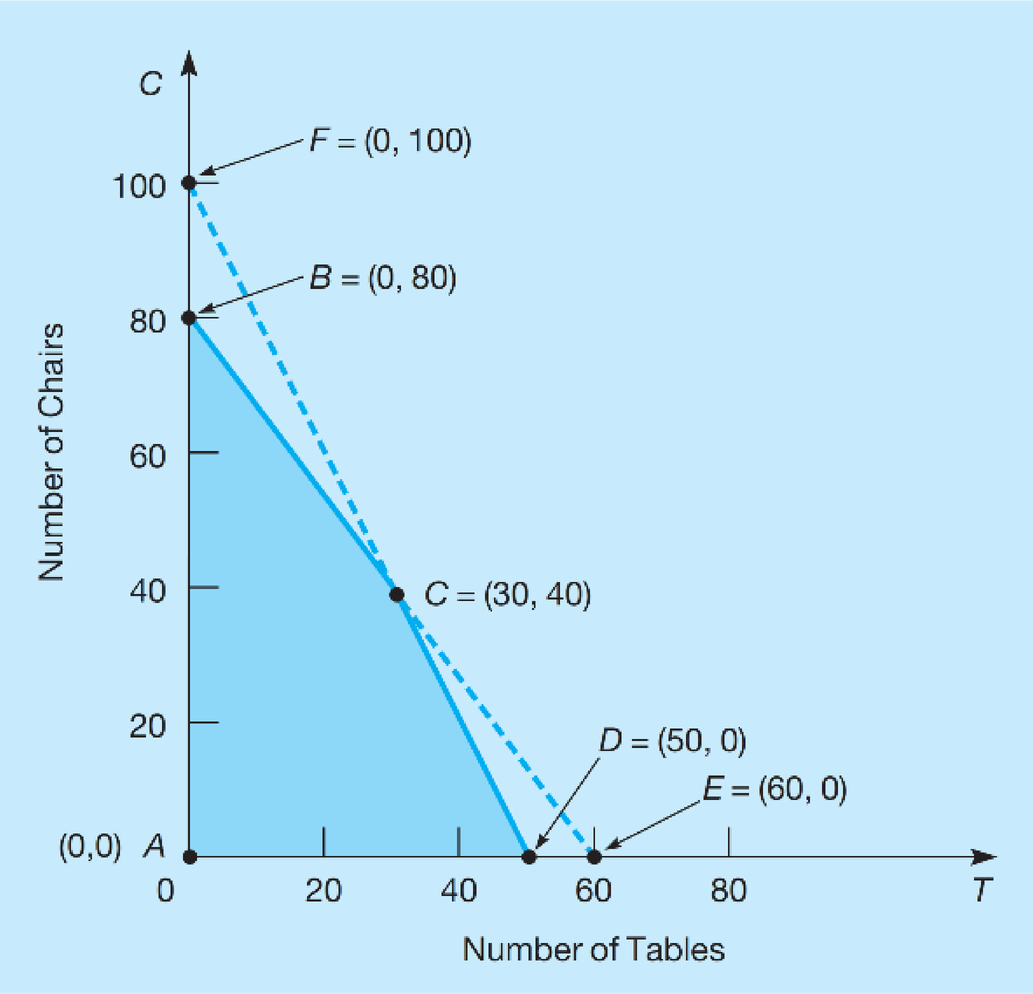
C_j → ↓	SOLUTION MIX	\$70 T	\$50 C	\$0 S_1	\$0 S_2	QUANTITY (RHS)
\$0	S_1	②	1	1	0	100 ← Pivot row
\$0	S_2	4	3	0	1	240
	Z_j	\$0	\$0	\$0	\$0	\$0
	$C_j - Z_j$	\$70	\$50	\$0	\$0	

Pivot number

Pivot column

The Second Simplex Tableau (4 of 10)

FIGURE M7.2 Graph of the Flair Furniture Company Problem



The Second Simplex Tableau (5 of 10)

- Step 3 – Compute the replacement for the pivot row

$$\frac{2}{2} = 1 \qquad \frac{1}{2} = 0.5 \qquad \frac{0}{2} = 0 \qquad \frac{100}{2} = 50$$

- The entire pivot row

C_j	SOLUTION MIX	T	C	S_1	S_2	QUANTITY
$70	T	1	0.5	0.5	0	50

The Second Simplex Tableau (6 of 10)

- Step 4 – Compute new values for the S_2 row

Number in New S_2 Row	=	Number in Old S_2 Row	−	Number Below Pivot Number	×	Corresponding Number in the New T Row
0	=	4	−	(4)	×	(1)
1	=	3	−	(4)	×	(0.5)
−2	=	0	−	(4)	×	(0.5)
1	=	1	−	(4)	×	(0)
40	=	240	−	(4)	×	(50)

The Second Simplex Tableau (7 of 10)

- The second tableau

C_j	SOLUTION MIX	T	C	S_1	S_2	QUANTITY
$70	T	1	0.5	0.5	0	50
$0	S_2	0	1	−2	1	40

The Second Simplex Tableau (8 of 10)

- Step 5 – Introduce the effect of the objective function

Z_j (for T column) = (\$70)(1) + (\$0)(0)= \$70

Z_j (for C column) = (\$70)(0.5) + (\$0)(1) = \$35

Z_j (for S_1 column) = (\$70)(0.5) + (\$0)(−2) = \$35

Z_j (for S_2 column) = (\$70)(0) + (\$0)(1) = \$0

Z_j (for total profit) = (\$70)(50) + (\$0)(40) = \$3,500

The Second Simplex Tableau (9 of 10)

- Step 5 – Introduce the effect of the objective function

TABLE M7.4 Completed Second Simplex Tableau for Flair Furniture

C_j →		$70	$50	$0	$0	
↓	**SOLUTION MIX**	***T***	***C***	S_1	S_2	**QUANTITY**
$70	*T*	1	0.5	0.5	0	50
$0	S_2	0	1	−2	1	40
	Z_j	$70	$35	$35	$0	$3,500
	$C_j - Z_j$	$0	$15	−$35	$0	

The Second Simplex Tableau (10 of 10)

- $C_j - Z_j$ numbers represent net profit at present production mix

	COLUMN			
	T	***C***	S_1	S_2
C_j for column	\$70	\$50	\$0	\$0
Z_j for column	\$70	\$35	\$35	\$0
$C_j - Z_j$ for column	\$0	\$15	−\$35	\$0

Interpreting the Second Tableau

- Current solution and resources
  - Production of 50 tables (T) and 0 chairs (C)
  - Profit = $3,500
  - T is a basic variable, C nonbasic
  - Slack variable $S_2 = 40$, S_1 nonbasic
- Substitution rates
  - Marginal rates of substitution
  - Negative substitution rate
    - If 1 unit of a column variable is added to the solution, the value of the corresponding solution (or row) variable will increase
  - Positive substitution rate
    - If 1 unit of the column variable is added to the solution, the row variable will decrease

Developing the Third Tableau (1 of 8)

- Not all numbers in the $C_j - Z_j$ row are 0 or negative
  - Previous solution is not optimal
  - Repeat the five simplex steps

- Step 1 – Variable *C* will enter the solution
  - Largest $C_j - Z_j$ value of 15

Developing the Third Tableau (2 of 8)

- Step 2 – Identifying the pivot row

$$\text{For the } T \text{ row: } \frac{50}{0.5} = 100 \text{ chairs} \qquad \text{For the } S_2 \text{ row: } \frac{40}{1} = 40 \text{ chairs}$$

TABLE M7.5 Pivot Row, Pivot Column, and Pivot Number Identified in the Second Simplex Tableau

C_j →		$70	$50	$0	$0	
↓	**SOLUTION MIX**	**T**	**C**	**S_1**	**S_2**	**QUANTITY**
$70	T	1	0.5	0.5	0	50
$0	S_2	0	①	−2	1	40
	Z_j	$70	$35	$35	$0	$3,500
	$C_j - Z_j$	$0	$15	−$35	$0	

← Pivot row

Pivot number

Pivot column

Developing the Third Tableau (3 of 8)

- Step 3 – The pivot row is replaced

$$\frac{0}{1} = 0 \qquad \frac{1}{1} = 1 \qquad \frac{-2}{1} = -2 \qquad \frac{1}{1} = 1 \qquad \frac{40}{1} = 40$$

- The new C row

C_j	SOLUTION MIX	T	C	S_1	S_2	QUANTITY
\$50	C	0	1	−2	1	40

Developing the Third Tableau (4 of 8)

- Step 4 – New values for the *T* row

(Number in new *T* row)	=	(Number in old *T* row)	−	[(Number above pivot number)	×	(Corresponding number in new *C* row)]
1	=	1	−	(0.5)	×	(0)
0	=	0.5	−	(0.5)	×	(1)
1.5	=	0.5	−	(0.5)	×	(−2)
−0.5	=	0	−	(0.5)	×	(1)
30	=	50	−	(0.5)	×	(40)

C_j	SOLUTION MIX	T	C	S_1	S_2	QUANTITY
\$70	*T*	1	0	1.5	−0.5	30
\$50	*C*	0	1	−2	1	40

Developing the Third Tableau (5 of 8)

- Step 5 – Calculate the Z_j and $C_j - Z_j$ rows

Z_j (for T column) = (\$70)(1) + (\$50)(0) = \$70

Z_j (for C column) = (\$70)(0) + (\$50)(1) = \$50

Z_j (for S_1 column) = (\$70)(1.5) + (\$50)(−2) = \$5

Z_j (for S_2 column) = (\$70)(−0.5) + (\$50)(1) = \$15

Z_j (for total profit) = (\$70)(30) + (\$50)(40) = \$4,100

	COLUMN			
	T	***C***	**S_1**	**S_2**
C_j for column	\$70	\$50	\$0	\$0
Z_j for column	\$70	\$50	\$5	\$15
$C_j - Z_j$ for column	\$0	\$0	−\$5	−\$15

Developing the Third Tableau (6 of 8)

- Step 5 – Calculate the Z_j and $C_j - Z_j$ rows

TABLE M7.6 Final Simplex Tableau for the Flair Furniture Problem

C_j →		**$70**	**$50**	**$0**	**$0**	
↓	**SOLUTION MIX**	***T***	***C***	S_1	S_2	**QUANTITY**
$70	*T*	1	0	1.5	−0.5	30
$50	*C*	0	1	−2	1	40
	Z_j	$70	$50	$5	$15	$4,100
	$C_j - Z_j$	$0	$0	−$5	−$15	

Developing the Third Tableau (7 of 8)

Since every number in the tableau's $C_j - Z_j$ row is 0 or negative, an optimal solution has been reached

TABLE M7.6 Final Simplex Tableau for the Flair Furniture Problem

C_j →		$70	$50	$0	$0	
↓	**SOLUTION MIX**	T	C	S_1	S_2	**QUANTITY**
$70	T	1	0	1.5	−0.5	30
$0	C	0	1	−2	1	40
	Z_j	$70	$50	$5	$15	$4,100
	$C_j - Z_j$	$0	$0	−$5	−$15	

Developing the Third Tableau (8 of 8)

- Verifying the solution,

First constraint: $2T + 1C \leq 100$ painting dept hours

$2(30) + 1(40) \leq 100$

$100 \leq 100$ ✓

Second constraint: $4T + 3C \leq 240$ carpentry dept hours

$4(30) + 3(40) \leq 240$

$240 \leq 240$ ✓

Objective function profit = $\$70T + \$50C$

$= \$70(30) + \$50(40)$

$= \$4{,}100$

Review of Procedures (1 of 3)

I. Formulate the LP problem's objective function and constraints.

II. Add slack variables to each less-than-or-equal-to constraint and to the problem's objective function.

III. Develop an initial simplex tableau with slack variables in the basis and the decision variables set equal to 0. Compute the Z_j and $C_j - Z_j$ values for this tableau.

Review of Procedures (2 of 3)

IV. Follow these five steps until an optimal solution has been reached:

1. Choose the variable with the greatest positive $C_j - Z_j$ to enter the solution. This is the pivot column.
2. Determine the solution mix variable to be replaced and the pivot row by selecting the row with the smallest (nonnegative) ratio of the quantity-to-pivot column substitution rate. This row is the pivot row.
3. Calculate the new values for the pivot row.

Review of Procedures (3 of 3)

4. Calculate the new values for the other row(s).
5. Calculate the Z_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers greater than 0, return to step 1. If there are no $C_j - Z_j$ numbers that are greater than 0, an optimal solution has been reached.

Surplus and Artificial Variables (1 of 3)

- Conversions are made for ≥ and = constraints
- For **Surplus Variables,**

$$\text{Constraint 1: } 5X_1 + 10X_2 + 8X_3 \geq 210$$

$$\text{Rewritten: } 5X_1 + 10X_2 + 8X_3 - S_1 = 210$$

For $X_1 = 20$, $X_2 = 8$, and $X_3 = 5$,

$$\begin{aligned} 5(20) + 10(8) + 8(5) - S_1 &= 210 \\ -S_1 &= 210 - 220 \\ S_1 &= 10 \text{ surplus units} \end{aligned}$$

Surplus and Artificial Variables (2 of 3)

- If X_1 and $X_2 = 0$, then the S_1 variable is negative, which violates the nonnegative condition
- An **artificial variable** A_1 is added to resolve this problem

Surplus and Artificial Variables (3 of 3)

- Conversions are made for ≥ and = constraints
- For equalities, add only an artificial variable

Constraint 2: $25X_1 + 30X_2 = 900$

Constraint 2 completed: $25X_1 + 30X_2 + A_2 = 900$

Artificial variables have no physical meaning and drop out of the solution mix before the final tableau if a feasible solution exists

Surplus and Artificial Variables in the Objective Function

- Assign a very high-cost \$*M* to artificial variables in the objective function to force them out before the final solution is reached

$$\text{Minimize cost} = \$5X_1 + \$9X_2 + \$7X_3$$

$$\text{Minimize cost} = \$5X_1 + \$9X_2 + \$7X_3 + \$0S_1 + \$MA_1 + \$MA_2$$

$$\text{subject to} \quad 5X_1 + 10X_2 + 8X_3 - 1S_1 + 1A_1 + 0A_2 = 210$$

$$25X_1 + 30X_2 + 0X_3 + 0S_1 + 0A_1 + 1A_2 = 900$$

Solving Minimization Problems

- **The Muddy River Chemical Company**

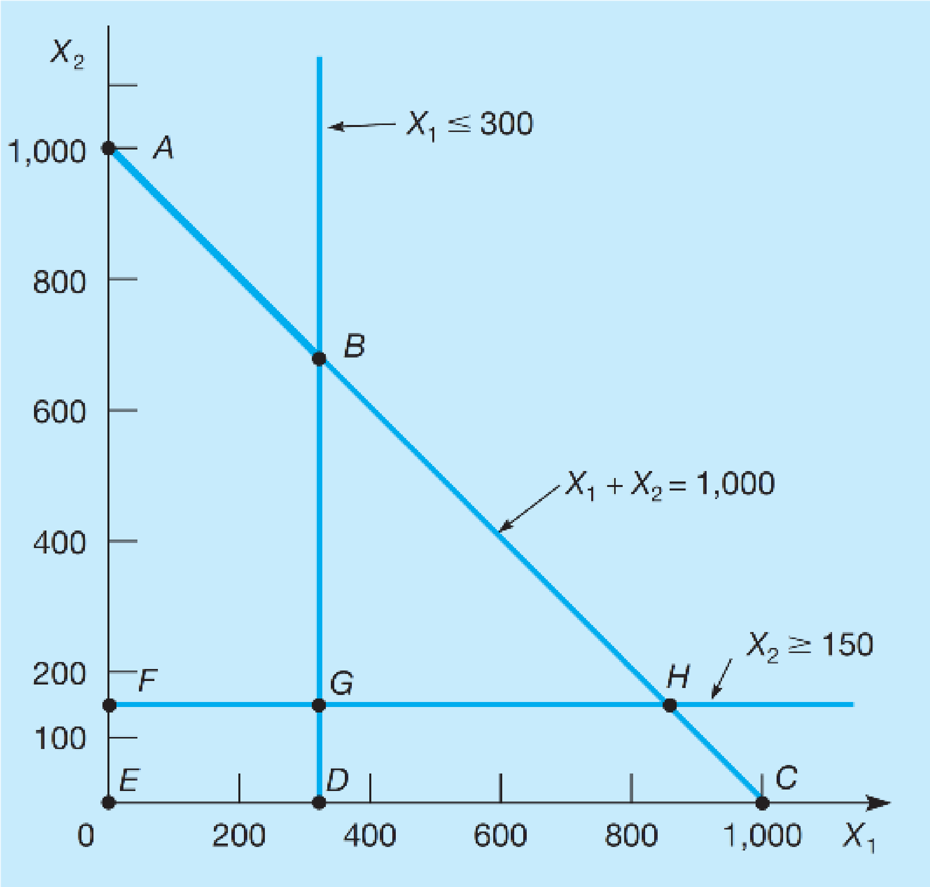
$$
\begin{aligned}
\text{Minimize cost} &= \$5X_1 + \$6X_2 \\
\text{subject to} \quad X_1 + X_2 &= 1{,}000 \text{ lb} \\
X_1 &\leq 300 \text{ lb} \\
X_2 &\geq 150 \text{ lb} \\
X_1, X_2 &\geq 0
\end{aligned}
$$

where

X_1 = number of pounds of phosphate
X_2 = number of pounds of potassium

Graphical Analysis

FIGURE M7.3 Muddy River Chemical Corporation's Feasible Region



Converting the Constraints and Objective Function

$$\begin{array}{lcrcrcrcrcrcrl}
\text{Minimize cost} & = & \$5X_1 & + & \$6X_2 & + & \$0S_1 & + & \$0S_2 & + & \$MA_1 & + & \$MA_2 & \\
\text{subject to} & & 1X_1 & + & 1X_2 & + & 0S_1 & + & 0S_2 & + & 1A_1 & + & 0A_2 & = 1{,}000 \\
 & & 1X_1 & + & 0X_2 & + & 1S_1 & + & 0S_2 & + & 0A_1 & + & 0A_2 & = 300 \\
 & & 0X_1 & + & 1X_2 & + & 0S_1 & - & 1S_2 & + & 0A_1 & + & 1A_2 & = 150 \\
 & & & & & & & & & & \multicolumn{3}{r}{X_1,\, X_2,\, S_1,\, S_2,\, A_1,\, A_2} & \geq 0
\end{array}$$

Rules of the Simplex Method for Minimization Problems

1. Choose the variable with a negative $C_j - Z_j$ that indicates the largest decrease in cost to enter the solution. The corresponding column is the pivot column.
2. Determine the row to be replaced by selecting the one with the smallest (nonnegative) quantity-to-pivot column substitution rate ratio. This is the pivot row.
3. Calculate new values for the pivot row.
4. Calculate new values for the other rows.
5. Calculate the Z_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers less than 0, return to step 1.

First Simplex Tableau (1 of 4)

C_j	SOLUTION MIX	X_1	X_2	S_1	S_2	A_1	A_2	QUANTITY
$\$M$	A_1	1	1	0	0	1	0	1,000
$\$0$	S_1	1	0	1	0	0	0	300
$\$M$	A_2	0	1	0	−1	0	1	150

Z_j (for X_1 column) $= \$M(1) + \$0(1) + \$M(0) = \$M$

Z_j (for X_2 column) $= \$M(1) + \$0(0) + \$M(1) = \$2M$

Z_j (for S_1 column) $= \$M(0) + \$0(1) + \$M(0) = \$0$

Z_j (for S_2 column) $= \$M(0) + \$0(0) + \$M(-1) = -\$M$

Z_j (for A_1 column) $= \$M(1) + \$0(0) + \$M(0) = \$M$

Z_j (for A_2 column) $= \$M(0) + \$0(0) + \$M(1) = \$M$

Z_j (for total cost) $= \$M(1{,}000) + \$0(300) + \$M(150) = \$1{,}150M$

Pearson

First Simplex Tableau (2 of 4)

	COLUMN					
	X_1	X_2	S_1	S_2	A_1	A_2
C_j for column	\$5	\$6	\$0	\$0	\$$M$	\$$M$
Zj for column	\$$M$	\$2M	\$0	−\$$M$	\$$M$	\$$M$
$C_j - Z_j$ for column	−\$$M$ + \$5	−\$2M + \$6	\$0	\$$M$	\$0	\$0

Pearson

First Simplex Tableau (3 of 4)

TABLE M7.7 Initial Simplex Tableau for the Muddy River Chemical Corporation Problem

C_j		$\$5$	$\$6$	$\$0$	$\$0$	$\$M$	$\$M$	
	SOLUTION MIX	X_1	X_2	S_1	S_2	A_1	A_2	**QUANTITY**
$\$M$	A_1	1	1	0	0	1	0	1,000
$\$0$	S_1	1	0	1	0	0	0	300
$\$M$	A_2	0	1	0	−1	0	1	150
	Z_j	$\$M$	$\$2M$	0	$-\$M$	$\$M$	$\$M$	$\$1,150M$
	$C_j - Z_j$	$-\$M + \$5$	$-\$2M + \$6$	$\$0$	$\$M$	$\$0$	$\$0$	

Pivot row

Pivot number

Pivot column

First Simplex Tableau (4 of 4)

In vector format

$$\begin{bmatrix} X_1 \\ X_2 \\ S_1 \\ S_2 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \\ 1{,}000 \\ 150 \end{bmatrix}$$

Developing a Second Tableau (1 of 3)

$$\text{For the } A_1 \text{ row} = \frac{1{,}000}{1} = 1{,}000$$

$$\text{For the } S_1 \text{ row} = \frac{300}{0} \quad \text{(undefined)}$$

$$\text{For the } A_2 \text{ row} = \frac{150}{0} \quad \text{(undefined)}$$

A_1 Row	S_1 Row
$1 = 1 - (1)(0)$	$1 = 1 - (0)(0)$
$0 = 1 - (1)(1)$	$0 = 0 - (0)(1)$
$0 = 0 - (1)(0)$	$1 = 1 - (0)(0)$
$1 = 0 - (1)(-1)$	$0 = 0 - (0)(-1)$
$1 = 1 - (1)(0)$	$0 = 0 - (0)(0)$
$-1 = 0 - (1)(1)$	$0 = 0 - (0)(1)$
$\underline{850 = 1{,}000 - (1)(150)}$	$\underline{300 = 300 - (0)(150)}$

Developing a Second Tableau (2 of 3)

Z_j (for X_1) $= \$M(1) + \$0(1) + \$6(0) = \$M$

Z_j (for X_2) $= \$M(0) + \$0(0) + \$6(1) = \$6$

Z_j (for S_1) $= \$M(0) + \$0(1) + \$6(0) = \$0$

Z_j (for S_2) $= \$M(1) + \$0(0) + \$6(-1) = \$M - 6$

Z_j (for A_1) $= \$M(1) + \$0(0) + \$6(0) = \$M$

Z_j (for A_2) $= \$M(-1) + \$0(0) + \$6(1) = -\$M + 6$

Z_j (for total cost) $= \$M(850) + \$0(300) + \$6(150) = \$850M + 900$

	COLUMN					
	X_1	**X_2**	**S_1**	**S_2**	**A_1**	**A_2**
C_j for column	\$5	\$6	\$0	\$0	$\$M$	$\$M$
Zj for column	$\$M$	\$6	\$0	$\$M - 6$	$\$M$	$-\$M + 6$
$C_j - Z_j$ for column	$-\$M + \$5$	\$0	\$0	$-\$M + 6$	\$0	$\$2M - 6$

Developing a Second Tableau (3 of 3)

TABLE M7.8 Second Simplex Tableau for the Muddy River Chemical Corporation Problem

C_j →		$5	$6	$0	$0	$M	$M	
↓	**SOLUTION MIX**	X_1	X_2	S_1	S_2	A_1	A_2	**QUANTITY**
$M	A_1	1	0	0	1	1	−1	850
$0	S_1	1	0	1	0	0	0	300 ← Pivot row
$6	X_2	0	1	0	−1	0	1	150
	Z_j	$M	$6	$0	$M − 6	$M	−$M + 6	$850M + $900
	$C_j - Z_j$	−$M + $5	$0	$0	−$M + 6	$0	$2M − 6	

Pivot number

Pivot column

Developing a Third Tableau (1 of 3)

$\underline{A_1 \text{ Row}}$	$\underline{X_2 \text{ Row}}$
$0 = 1 - (1)(1)$	$0 = 0 - (0)(1)$
$0 = 0 - (1)(0)$	$1 = 1 - (0)(0)$
$-1 = 0 - (1)(1)$	$0 = 0 - (0)(1)$
$1 = 1 - (1)(0)$	$-1 = -1 - (0)(0)$
$1 = 1 - (1)(0)$	$0 = 0 - (0)(0)$
$-1 = -1 - (1)(0)$	$1 = 1 - (0)(0)$
$\underline{550 = 850 - (1)(150)}$	$\underline{150 = 150 - (0)(300)}$

Developing a Third Tableau (2 of 3)

$Z_j \text{ (for } X_1) = \$M(0) + \$5(1) + \$6(0) = \$5$

$Z_j \text{ (for } X_2) = \$M(0) + \$5(0) + \$6(1) = \$6$

$Z_j \text{ (for } S_1) = \$M(-1) + \$5(1) + \$6(0) = -\$M + 5$

$Z_j \text{ (for } S_2) = \$M(1) + \$5(0) + \$6(-1) = \$M - 6$

$Z_j \text{ (for } A_1) = \$M(1) + \$5(0) + \$6(0) = \$M$

$Z_j \text{ (for } A_2) = \$M(-1) + \$5(0) + \$6(1) = -\$M + 6$

$Z_j \text{ (for total cost)} = \$M(550) + \$5(300) + \$6(150) = \$550M + 2{,}400$

	COLUMN					
	X_1	X_2	S_1	S_2	A_1	A_2
C_j for column	\$5	\$6	\$0	\$0	$\$M$	$\$M$
Z_j for column	\$5	\$6	$-\$M + 5$	$\$M - 6$	$\$M$	$-\$M + 6$
$C_j - Z_j$ for column	\$0	\$0	$\$M - 5$	$-\$M + 6$	\$0	$\$2M - 6$

Developing a Third Tableau (3 of 3)

TABLE M7.9 Third Simplex Tableau for the Muddy River Chemical Corporation Problem

C_j		\$5	\$6	\$0	\$0	\$$M$	\$$M$	
	SOLUTION MIX	X_1	X_2	S_1	S_2	A_1	A_2	**QUANTITY**
\$$M$	A_1	0	0	−1	1	1	−1	550
\$5	X_1	1	0	1	0	0	0	300
\$6	X_2	0	1	0	−1	0	1	150
	Z_j	\$5	\$6	−\$$M$ + 5	\$$M$ − 6	\$$M$	−\$$M$ + 6	\$550M + 2,400
	$C_j - Z_j$	\$0	\$0	\$$M$ − 5	−\$$M$ + 6	\$0	\$2M − 6	

Pivot row

Pivot number

Pivot column

Developing a Fourth Tableau (1 of 3)

$$\text{For the } A_1 \text{ row} = \frac{550}{1} = 550$$

$$\text{For the } X_1 \text{ row} = \frac{300}{0} \quad \text{(undefined)}$$

$$\text{For the } X_2 \text{ row} = \frac{150}{-1} \quad \text{(negative)}$$

X_1 Row	X_2 Row
$1 = 1 - (0)(0)$	$0 = 0 - (-1)(0)$
$0 = 0 - (0)(0)$	$1 = 1 - (-1)(0)$
$1 = 1 - (0)(-1)$	$-1 = 0 - (-1)(-1)$
$0 = 0 - (0)(1)$	$0 = -1 - (-1)(1)$
$0 = 0 - (0)(1)$	$1 = 0 - (-1)(1)$
$0 = 0 - (0)(-1)$	$0 = 1 - (-1)(-1)$
$\underline{300 = 300 - (0)(550)}$	$\underline{700 = 150 - (-1)(550)}$

Developing a Fourth Tableau (2 of 3)

Z_j (for X_1) = \$0(0) + \$5(1) + \$6(0) = \$5

Z_j (for X_2) = \$0(0) + \$5(0) + \$6(1) = \$6

Z_j (for S_1) = \$0(−1) + \$5(1) + \$6(−1) = −\$1

Z_j (for S_2) = \$0(1) + \$5(0) + \$6(0) = \$0

Z_j (for A_1) = \$0(1) + \$5(0) + \$6(1) = \$6

Z_j (for A_2) = \$0(−1) + \$5(0) + \$6(0) = \$0

Z_j (for total cost) = \$0(550) + \$5(300) + \$6(700) = \$5,700

	COLUMN					
	X_1	X_2	S_1	S_2	A_1	A_2
C_j for column	\$5	\$6	\$0	\$0	\$$M$	\$$M$
Z_j for column	\$5	\$6	−\$1	\$0	\$6	\$0
$C_j - Z_j$ for column	\$0	\$0	\$1	\$0	\$$M$ − 6	\$$M$

Developing a Fourth Tableau (3 of 3)

TABLE M7.10 Fourth and Optimal Solution to the Muddy River Chemical Corporation Problem

C_j		\$5	\$6	\$0	\$0	$\$M$	$\$M$	
	SOLUTION MIX	X_1	X_2	S_1	S_2	A_1	A_2	**QUANTITY**
\$0	S_2	0	0	−1	1	1	−1	550
\$5	X_1	1	0	1	0	0	0	300
\$6	X_2	0	1	−1	0	1	0	700
	Z_j	\$5	\$6	−\$1	\$0	\$6	\$0	\$5,700
	$C_j - Z_j$	\$0	\$0	\$1	\$0	$\$M - 6$	$\$M$	

Review of Procedures (1 of 3)

I. Formulate the LP problem's objective function and constraints.

II. Include slack variables in each less-than-or-equal-to constraint, artificial variables in each equality constraint, and both surplus and artificial variables in each greater-than-or-equal-to constraint. Then add all of these variables to the problem's objective function.

III. Develop an initial simplex tableau with artificial and slack variables in the basis and the other variables set equal to 0. Compute the Z_j and $C_j - Z_j$ values for this tableau.

Review of Procedures (2 of 3)

IV. Follow these five steps until an optimal solution has been reached:

1. Choose the variable with the negative $C_j - Z_j$ indicating the greatest improvement to enter the solution. This is the pivot column.
2. Determine the row to be replaced by selecting the one with the smallest (nonnegative) quantity-to-pivot column substitution rate ratio. This is the pivot row.
3. Calculate the new values for the pivot row.
4. Calculate the new values for the other row(s).

Review of Procedures (3 of 3)

5. Calculate the Z_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers less than 0, return to step 1. If there are no $C_j - Z_j$ numbers that are less than 0, an optimal solution has been reached.

Special Cases (1 of 6)

- **Infeasibility**
  - **Infeasibility** occurs when there is no solution that satisfies all of the problem’s constraints

TABLE M7.11 Illustration of Infeasibility

C_j →		$5	$8	$0	$0	$M	$M	
↓	**SOLUTION MIX**	X_1	X_2	S_1	S_2	A_1	A_2	**QUANTITY**
$5	X_1	1	0	−2	3	−1	−0	200
$8	X_2	0	1	1	2	−2	0	100
$M	A_2	0	0	0	−1	−1	1	20
	Z_j	$5	$8	−$2	$31 − M	−$21 − M	$M	$1,800 + 20M
	$C_j - Z_j$	$0	$0	$2	$M − 31	$2M + 21	$0	

Special Cases (2 of 6)

- **Unbounded Solutions**
  - **Unboundedness** describes linear programs that do not have finite solutions

TABLE M7.12 Problem with an Unbounded Solution

C_j		$6	$9	$0	$0	
	SOLUTION MIX	X_1	X_2	S_1	S_2	QUANTITY
$9	X_1	−1	1	2	0	30
$0	S_2	−2	0	−1	1	10
	Z_j	−$9	$9	$18	$0	$270
	$C_j - Z_j$	$15	$0	−$18	$0	
		Pivot column				

Special Cases (3 of 6)

Negative ratios unacceptable

TABLE M7.12 Problem with an Unbounded Solution

C_j	SOLUTION MIX	$6 X_1	$9 X_2	$0 S_1	$0 S_2	QUANTITY
$9	X_1	−1	1	2	0	30
$0	S_2	−2	0	−1	1	10
	Z_j	−$9	$9	$18	$0	$270
	$C_j - Z_j$	$15	$0	−$18	$0	

Pivot column

Special Cases (4 of 6)

- **Degeneracy**
  - **Degeneracy** develops when three constraints pass through a single point

TABLE M7.13 Problem Illustrating Degeneracy

C_j		$5	$8	$2	$0	$0	$0	
	SOLUTION MIX	X_1	X_2	X_3	S_1	S_2	S_3	**QUANTITY**
$8	X_2	0.25	1	1	−2	0	0	10
$0	S_2	4	0	0.33	−1	1	0	20
$0	S_1	2	0	2	0.4	0	1	10
	Z_j	$2	$8	$8	$16	$0	$0	$80
	$C_j - Z_j$	$3	$0	−$6	−$16	$0	$0	

Pivot column

Special Cases (5 of 6)

Tie for the smallest ratio indicates degeneracy

C_j		\$5	\$8	\$2	\$0	\$0	\$0	
	SOLUTION MIX	X_1	X_2	X_3	S_1	S_2	S_3	**QUANTITY**
\$8	X_2	0.25	1	1	−2	0	0	10
\$0	S_2	4	0	0.33	−1	1	0	20
\$0	S_1	2	0	2	0.4	0	1	10
	Z_j	\$2	\$8	\$8	\$16	\$0	\$0	\$80
	$C_j - Z_j$	\$3	\$0	−\$6	−\$16	\$0	\$0	

Pivot column

Special Cases (6 of 6)

- **More Than One Optimal Solution**
  - If the $C_j - Z_j$ value is equal to 0 for a variable that is not in the solution mix, more than one optimal solution exists

TABLE M7.14 Problem with Alternate Optimal Solutions

$C_j \rightarrow$		**\$3**	**\$2**	**\$0**	**\$0**	
$\downarrow$	**SOLUTION MIX**	X_1	X_2	S_1	S_2	**QUANTITY**
\$2	X_2	1.5	1	1	0	6
\$0	S_2	1	0	0.5	1	3
	Z_j	\$3	\$2	\$2	\$0	\$12
	$C_j - Z_j$	\$0	\$0	−\$2	\$0	

Quantitative Analysis for Management

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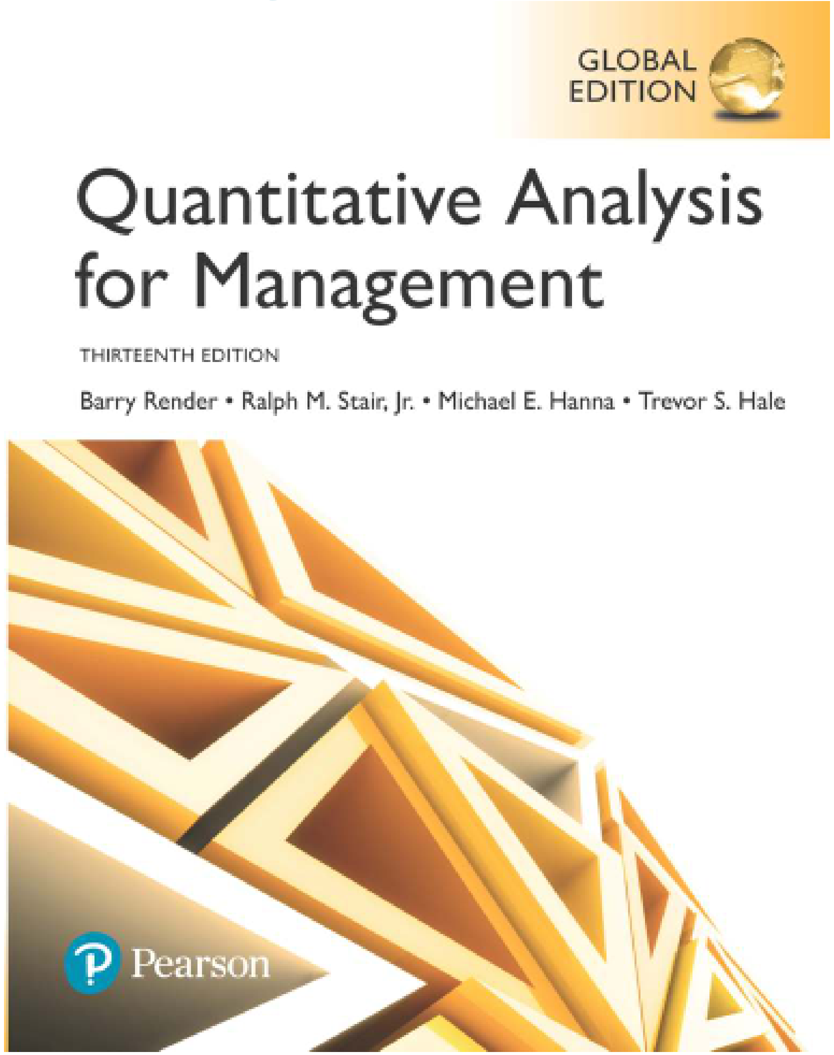


GLOBAL EDITION

Quantitative Analysis for Management

THIRTEENTH EDITION

Barry Render • Ralph M. Stair, Jr. • Michael E. Hanna • Trevor S. Hale

Pearson

Module 8

Transportation, Assignment, and Network Algorithms

Introduction

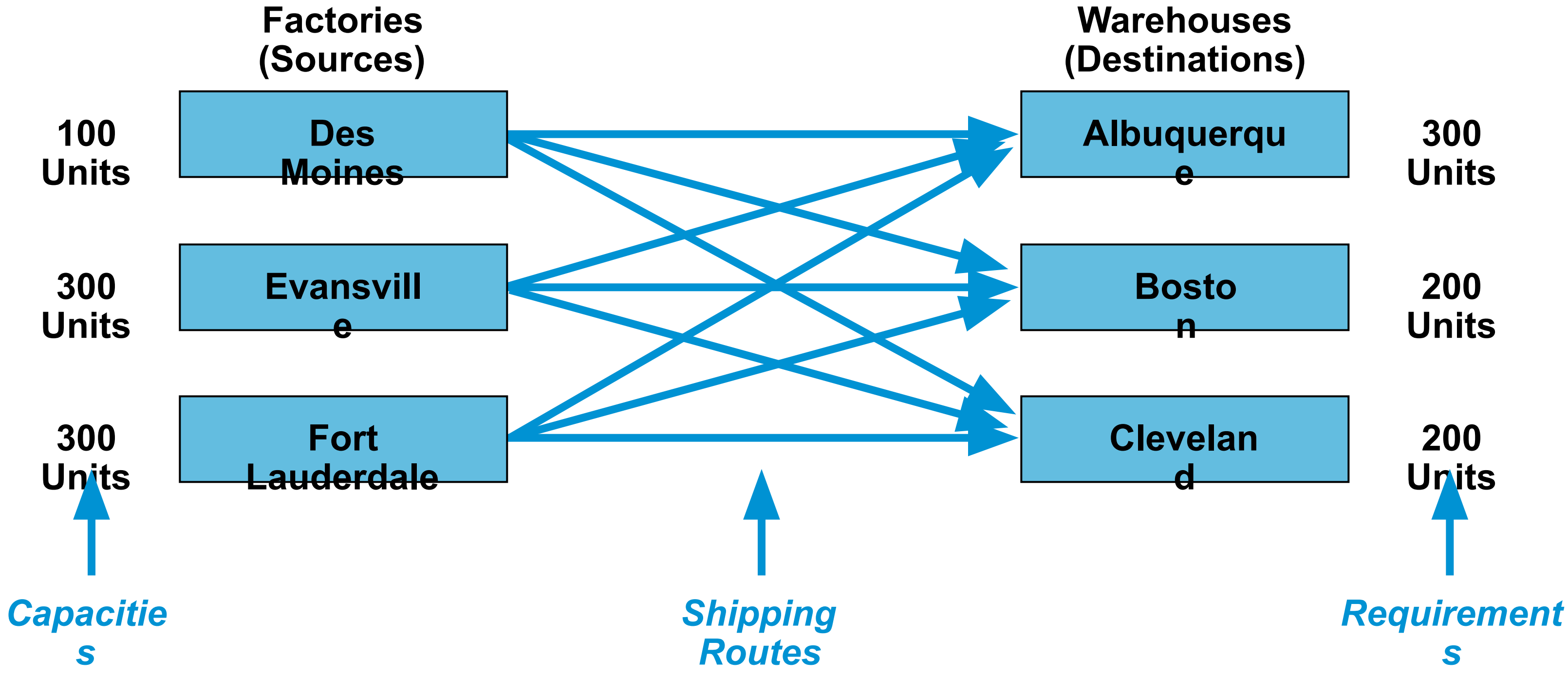
- In this chapter we will explore two special linear programming models
    - The transportation model
    - The assignment model
- Because of their structure, they can be solved more efficiently than the simplex method
- These problems are members of a category of LP techniques called ***network flow problems***

Introduction

- Transportation model
  - The *transportation problem* deals with the distribution of goods from several points of supply (*sources*) to a number of points of demand (*destinations*)
  - Usually we are given the capacity of goods at each source and the requirements at each destination
  - Typically the objective is to minimize total transportation and production costs

Introduction

- **Example of a transportation problem in a network format**



Factories (Sources)

Warehouses (Destinations)

100 Units — Des Moines

300 Units — Evansville

300 Units — Fort Lauderdale

Albuquerque — 300 Units

Boston — 200 Units

Cleveland — 200 Units

Capacities

Shipping Routes

Requirements

Figure 10.1

Introduction

- Assignment model
  - The *assignment problem* refers to the class of LP problems that involve determining the most efficient assignment of resources to tasks
  - The objective is most often to minimize total costs or total time to perform the tasks at hand
  - One important characteristic of assignment problems is that only one job or worker can be assigned to one machine or project

Introduction

- Special-purpose algorithms
    - Although standard LP methods can be used to solve transportation and assignment problems, special-purpose algorithms have been developed that are more efficient
    - They still involve finding and initial solution and developing improved solutions until an optimal solution is reached
    - They are fairly simple in terms of computation

Introduction

- **Streamlined versions of the simplex method are important for two reasons**
  1. **Their computation times are generally 100 times faster**
  2. **They require less computer memory (and hence can permit larger problems to be solved)**
- **Two common techniques for developing initial solutions are the northwest corner method and Vogel's approximation**
- **The initial solution is evaluated using either the stepping-stone method or the modified distribution (MODI) method**
- **We also introduce a solution procedure called the *Hungarian method*, *Flood's technique*, or the *reduced matrix method***

Setting Up a Transportation Problem

- **The Executive Furniture Corporation manufactures office desks at three locations: Des Moines, Evansville, and Fort Lauderdale**
- **The firm distributes the desks through regional warehouses located in Boston, Albuquerque, and Cleveland**
- **Estimates of the monthly production capacity of each factory and the desks needed at each warehouse are shown in Figure 10.1**

Setting Up a Transportation Problem

- Production costs are the same at the three factories so the only relevant costs are shipping from each *source* to each *destination*
- Costs are constant no matter the quantity shipped
- The transportation problem can be described as *how to select the shipping routes to be used and the number of desks to be shipped on each route so as to minimize total transportation cost*
- Restrictions regarding factory capacities and warehouse requirements must be observed

Setting Up a Transportation Problem

- The first step is setting up the transportation table
- Its purpose is to summarize all the relevant data and keep track of algorithm computations

Transportation costs per desk for Executive Furniture

TO FROM	ALBUQUERQUE	BOSTON	CLEVELAND
DES MOINES	$5	$4	$3
EVANSVILLE	$8	$4	$3
FORT LAUDERDALE	$9	$7	$5

Table 10.1

Setting Up a Transportation Problem

- Geographical locations of Executive Furniture's factories and warehouses

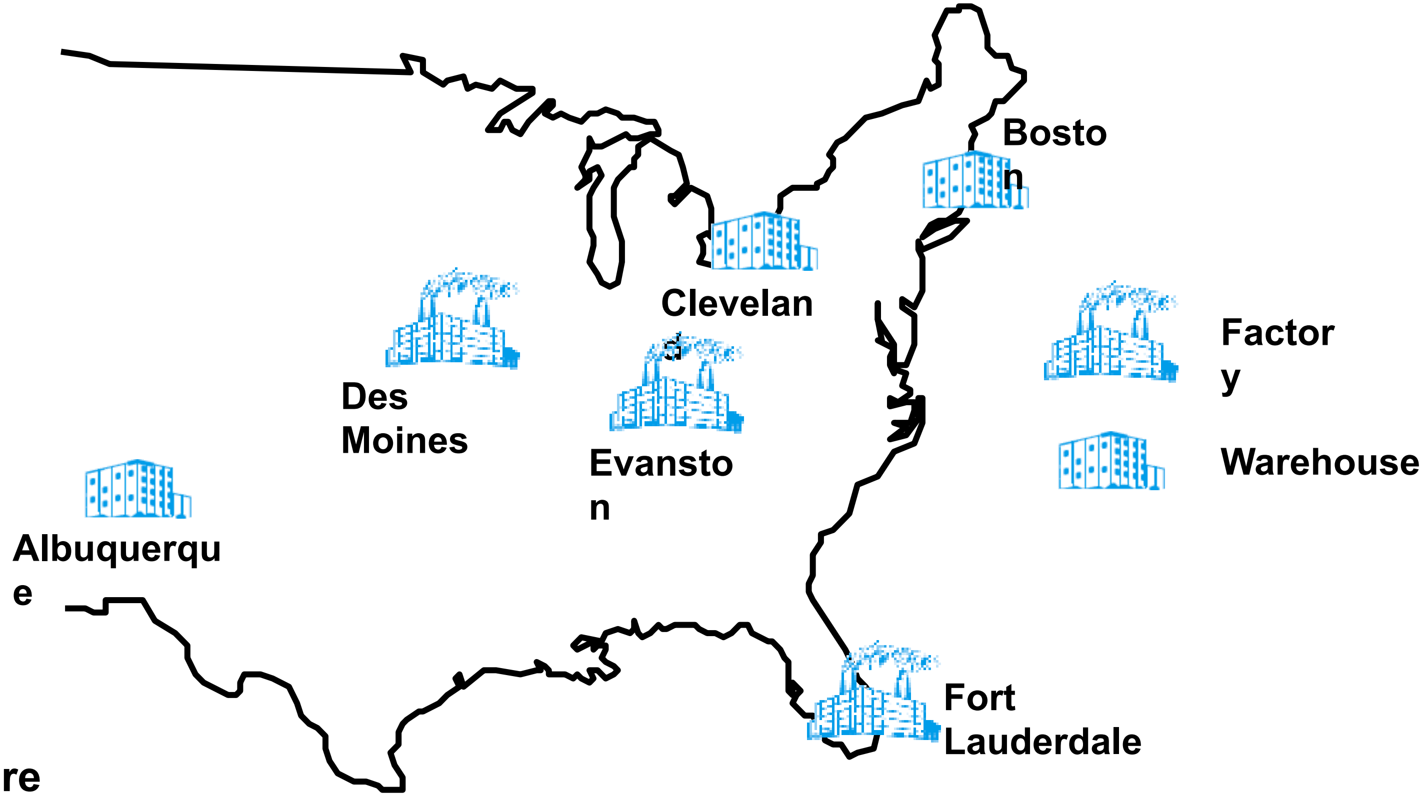


Figure 10.2

Setting Up a Transportation Problem

- **Transportation table for Executive Furniture**

FROM / TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	$5	$4	$3	100
EVANSVILLE FACTORY	$8	$4	$3	300
FORT LAUDERDALE FACTORY	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 10.2

Des Moines capacity constraint

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Setting Up a Transportation Problem

- In this table, total factory supply exactly equals total warehouse demand
- When equal demand and supply occur, a ***balanced problem*** is said to exist
- This is uncommon in the real world and we have techniques to deal with unbalanced problems

Developing an Initial Solution: Northwest Corner Rule

- **Once we have arranged the data in a table, we must establish an initial feasible solution**
- **One systematic approach is known as the *northwest corner rule***
- **Start in the upper left-hand cell and allocate units to shipping routes as follows**
  1. **Exhaust the supply (factory capacity) of each row before moving down to the next row**
  2. **Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column**
  3. **Check that all supply and demand requirements are met.**
- **In this problem it takes five steps to make the initial shipping assignments**

Developing an Initial Solution: Northwest Corner Rule

1. Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.

TO FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

2. **Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.**

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8 200	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

3. Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

TO FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8 200	$4 100	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

4. Assign 100 units from Fort Lauderdale to Boston. This fulfills Boston's demand and Fort Lauderdale still has 200 units available.

TO FROM	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	\$5 100	\$4	\$3	100
EVANSVILLE (*E*)	\$8 200	\$4 100	\$3	300
FORT LAUDERDALE (*F*)	\$9	\$7 100	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

5. **Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.**

Table 10.3

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8 200	$4 100	$3	300
FORT LAUDERDALE (*F*)	$9	$7 100	$5 200	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

- We can easily compute the cost of this shipping assignment

ROUTE						
FROM	TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

- This solution is feasible but we need to check to see if it is optimal

Vogel's Approximation Method: Another Way To Find An Initial Solution

- *Vogel's Approximation Method* (*VAM*) is not as simple as the northwest corner method, but it provides a very good initial solution, often one that is the *optimal* solution
- VAM tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative
- This is something that the northwest corner rule does not do
- To apply VAM, we first compute for each row and column the penalty faced if we should ship over the *second-best* route instead of the *least-cost* route

Vogel's Approximation Method

- The six steps involved in determining an initial VAM solution are illustrated below beginning with the same layout originally shown in Table 10.2

VAM Step 1. For each row and column of the transportation table, find the difference between the distribution cost on the *best* route in the row or column and the *second best* route in the row or column

  - This is the *opportunity cost* of not using the best route
  - Step 1 has been done in Table 10.11

Vogel's Approximation Method

- **Transportation table with VAM row and column differences shown**

	3	0	0	OPPORTUNITY COSTS	
FROM TO	*A*	*B*	*C*	TOTAL AVAILABLE	
D	100 $5	$4	$3	100	1
E	200 $8	100 $4	$3	300	1
F	$9	100 $7	200 $5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.11

Vogel's Approximation Method

VAM Step 2. identify the row or column with the greatest opportunity cost, or difference (column *A* in this example)

VAM Step 3.Assign as many units as possible to the lowest-cost square in the row or column selected

VAM Step 4. Eliminate any row or column that has been completely satisfied by the assignment just made by placing *X*s in each appropriate square

VAM Step 5. Recompute the cost differences for the transportation table, omitting rows or columns eliminated in the previous step

Vogel’s Approximation Method

- **VAM assignment with *D*’s requirements satisfied**

	~~3~~ 1	~~0~~ 3	~~0~~ 2	OPPORTUNITY COSTS	
TO FROM	*A*	*B*	*C*	TOTAL AVAILABLE	
D	$5 100	$4 X	$3 X	100	1
E	$8	$4	$3	300	1
F	$9	$7	$5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.12

Vogel's Approximation Method

VAM Step 6. Return to step 2 for the rows and columns remaining and repeat the steps until an initial feasible solution has been obtained

- In this case column *B* now has the greatest difference, 3
- We assign 200 units to the lowest-cost square in the column, *EB*
- We recompute the differences and find the greatest difference is now in row *E*
- We assign 100 units to the lowest-cost square in the column, *EC*

Vogel’s Approximation Method

- **Second VAM assignment with *B*’s requirements satisfied**

	~~3~~ 1	~~0~~ 3	~~0~~ 2	OPPORTUNITY COSTS	
TO FROM	*A*	*B*	*C*	TOTAL AVAILABLE	
D	$5 100	$4 X	$3 X	100	1
E	$8	$4 200	$3	300	1
F	$9	$7 X	$5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 10.13

Vogel’s Approximation Method

- **Third VAM assignment with *E*’s requirements satisfied**

TO FROM	*A*	*B*	*C*	TOTAL AVAILABLE
D	$5 100	$4 X	$3 X	100
E	$8 X	$4 200	$3 100	300
F	$9	$7 X	$5	300
TOTAL REQUIRED	300	200	200	700

Table 10.14

Vogel’s Approximation Method

- **Final assignments to balance column and row requirements**

TO FROM	A	B	C	TOTAL AVAILABLE
D	$5 100	$4 X	$3 X	100
E	$8 X	$4 200	$3 100	300
F	$9 200	$7 X	$5 100	300
TOTAL REQUIRED	300	200	200	700

Table 10.15

Unbalanced Transportation Problems

- **In real-life problems, total demand is frequently not equal to total supply**
- **These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations***
- **If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created**
- **If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply**

Unbalanced Transportation Problems

- In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped
- Any units assigned to a dummy destination represent excess capacity
- Any units assigned to a dummy source represent unmet demand

Demand Less Than Supply

- Suppose that the Des Moines factory increases its rate of production from 100 to 250 desks
- The firm is now able to supply a total of 850 desks each period
- Warehouse requirements remain the same (700) so the row and column totals do not balance
- We add a dummy column that will represent a fake warehouse requiring 150 desks
- This is somewhat analogous to adding a slack variable
- We use the northwest corner rule and either stepping-stone or MODI to find the optimal solution

Demand Less Than Supply

- Initial solution to an unbalanced problem where demand is less than supply

TO FROM	*A*	*B*	*C*	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	$5 250	$4	$3	0	250
E	$8 50	$4 200	$3 50	0	300
F	$9	$7	$5 150	0 150	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Table 10.16

New Des Moines capacity

Demand Greater than Supply

- The second type of unbalanced condition occurs when total demand is greater than total supply
- In this case we need to add a dummy row representing a fake factory
- The new factory will have a supply exactly equal to the difference between total demand and total real supply
- The shipping costs from the dummy factory to each destination will be zero

Demand Greater than Supply

- **Unbalanced transportation table for Happy Sound Stereo Company**

TO FROM	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	$6	$4	$9	200
PLANT *X*	$10	$5	$8	175
PLANT *Y*	$12	$7	$6	75
WAREHOUSE DEMAND	250	100	150	450 500

Table 10.17

Totals do not balance

Demand Greater than Supply

- **Initial solution to an unbalanced problem in which demand is greater than supply**

FROM / TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 $6	$4	$9	200
PLANT *X*	50 $10	100 $5	25 $8	175
PLANT *Y*	$12	$7	75 $6	75
PLANT *Y*	0	0	50 0	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

Table 10.18

Assignment Model Approach

- The second special-purpose LP algorithm is the assignment method
- Each assignment problem has associated with it a table, or matrix
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things we want them assigned to
- The numbers in the table are the costs associated with each particular assignment
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1

Assignment Model Approach

- The Fix-It Shop has three rush projects to repair
- They have three repair persons with different talents and abilities
- The owner has estimates of wage costs for each worker for each project
- The owner's objective is to assign the three project to the workers in a way that will result in the lowest cost to the shop
- Each project will be assigned exclusively to one worker

Assignment Model Approach

- **Estimated project repair costs for the Fix-It shop assignment problem**

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 10.26

Assignment Model Approach

- **Summary of Fix-It Shop assignment alternatives and costs**

PRODUCT ASSIGNMENT				
1	2	3	LABOR COSTS ($)	TOTAL COSTS ($)
Adams	Brown	Cooper	11 + 10 + 7	28
Adams	Cooper	Brown	11 + 12 + 11	34
Brown	Adams	Cooper	8 + 14 + 7	29
Brown	Cooper	Adams	8 + 12 + 6	26
Cooper	Adams	Brown	9 + 14 + 11	34
Cooper	Brown	Adams	9 + 10 + 6	25

Table 10.27

The Hungarian Method (Flood's Technique*)*

- The ***Hungarian method*** is an efficient method of finding the optimal solution to an assignment problem without having to make direct comparisons of every option
- It operates on the principle of ***matrix reduction***
- By subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of ***opportunity costs***
- Opportunity costs show the relative penalty associated with assigning any person to a project as opposed to making the ***best*** assignment
- We want to make assignment so that the opportunity cost for each assignment is zero

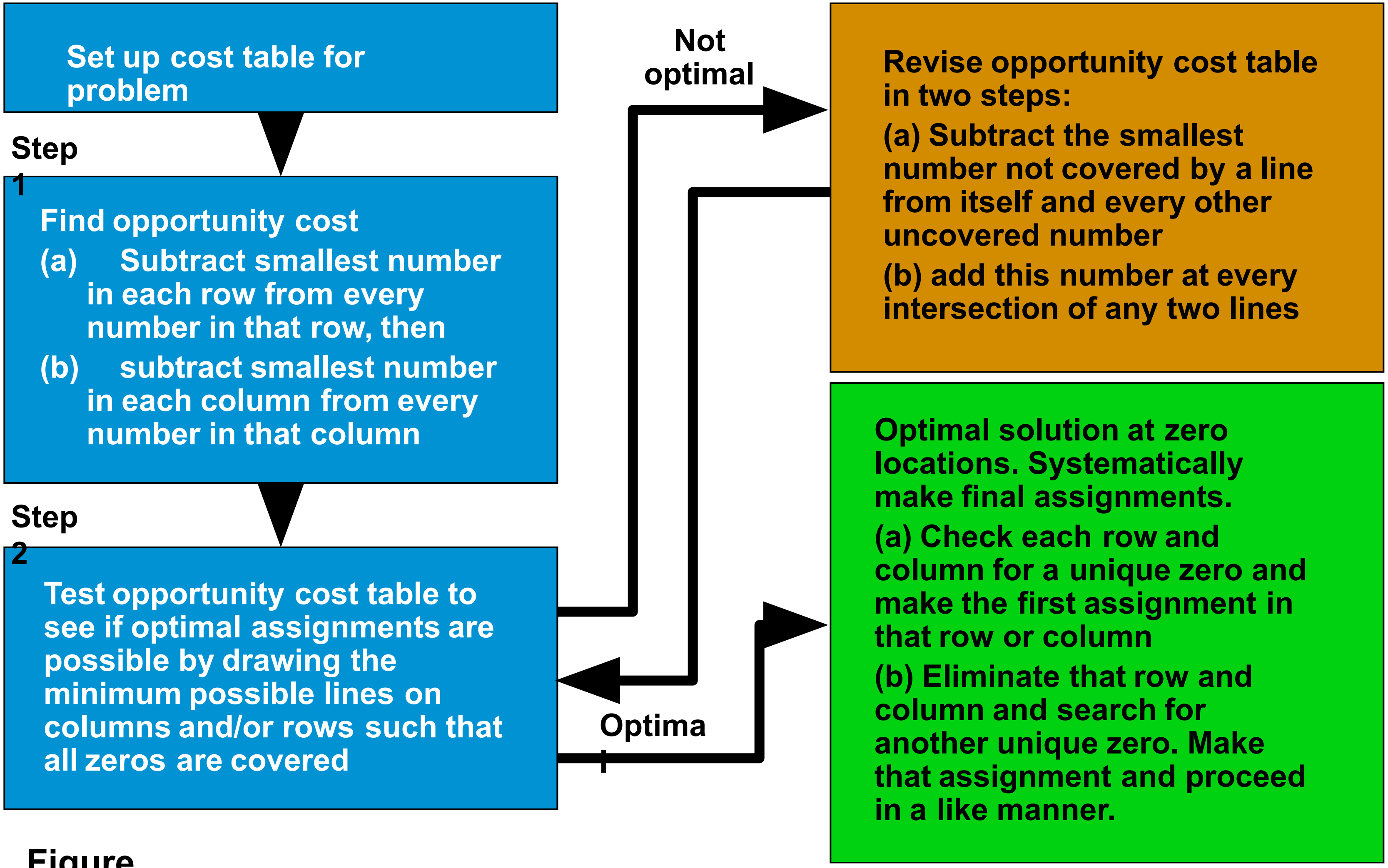
Three Steps of the Assignment Method

1. ***Find the opportunity cost table by*:**
   (a) Subtracting the smallest number in each row of the original cost table or matrix from every number in that row
   (b) Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column
2. ***Test the table resulting from step 1 to see whether an optimal assignment can be made* by drawing the minimum number of vertical and horizontal straight lines necessary to cover all the zeros in the table. If the number of lines is less than the number of rows or columns, proceed to step 3.**

Three Steps of the Assignment Method

3. *Revise the present opportunity cost table* by subtracting the smallest number not covered by a line from every other uncovered number. This same number is also added to any number(s) lying at the intersection of horizontal and vertical lines. Return to step 2 and continue the cycle until an optimal assignment is possible.

Steps in the Assignment Method



Set up cost table for problem

Step 1

Find opportunity cost

(a) Subtract smallest number in each row from every number in that row, then

(b) subtract smallest number in each column from every number in that column

Step 2

Test opportunity cost table to see if optimal assignments are possible by drawing the minimum possible lines on columns and/or rows such that all zeros are covered

Not optimal

Revise opportunity cost table in two steps:

(a) Subtract the smallest number not covered by a line from itself and every other uncovered number

(b) add this number at every intersection of any two lines

Optimal

Optimal solution at zero locations. Systematically make final assignments.

(a) Check each row and column for a unique zero and make the first assignment in that row or column

(b) Eliminate that row and column and search for another unique zero. Make that assignment and proceed in a like manner.

Figure 10.3

The Hungarian Method
(Flood's Technique)

- Step 1: Find the opportunity cost table
  - We can compute *row* opportunity costs and *column* opportunity costs
  - What we need is the *total* opportunity cost
  - We derive this by taking the row opportunity costs and subtract the smallest number in that column from each number in that column

The Hungarian Method (Flood's Technique)

- Cost of each person-project assignment

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 10.28

- Row opportunity cost table

	PROJECT		
PERSON	1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Table 10.29

- The opportunity cost of assigning Cooper to project 2 is $12 – $7 = $5

The Hungarian Method (Flood's Technique)

- We derive the total opportunity costs by taking the costs in Table 29 and subtract the smallest number in each column from each number in that column

- Row opportunity cost table

	PROJECT		
PERSON	1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Table 10.29

- Total opportunity cost table

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 10.30

The Hungarian Method (Flood's Technique)

- Step 2: Test for the optimal assignment
  - We want to assign workers to projects in such a way that the total labor costs are at a minimum
  - We would like to have a total assigned opportunity cost of zero
  - The test to determine if we have reached an optimal solution is simple
  - We find the *minimum* number of straight lines necessary to cover all the zeros in the table
  - If the number of lines equals the number of rows or columns, an optimal solution has been reached

The Hungarian Method (Flood's Technique)

- Test for optimal solution

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 10.31

Covering line 1

Covering line 2

- This requires only two lines to cover the zeros so the solution is not optimal

The Hungarian Method (Flood's Technique)

- Step 3: Revise the opportunity-cost table
  - We *subtract* the smallest number not covered by a line from all numbers not covered by a straight line
  - The same number is added to every number lying at the intersection of any two lines
  - We then return to step 2 to test this new table

The Hungarian Method (Flood's Technique)

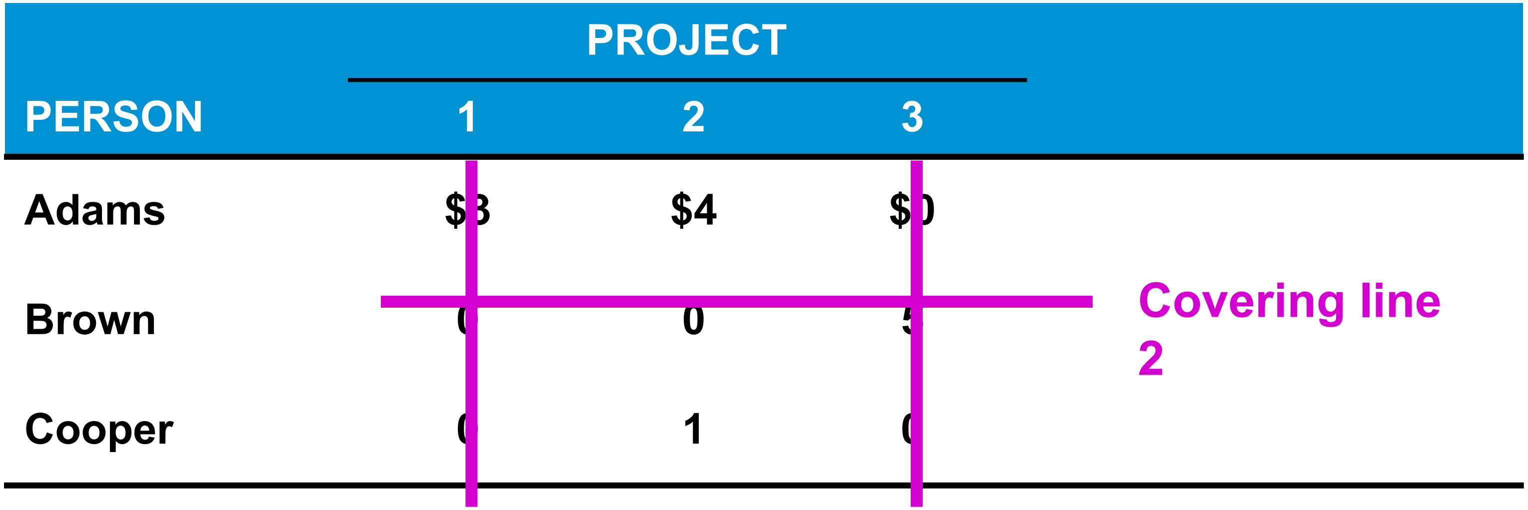
- Revised opportunity cost table (derived by subtracting 2 from each cell not covered by a line and adding 2 to the cell at the intersection of the lines)

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 10.32

The Hungarian Method
(Flood's Technique)

- Optimality test on the revised opportunity cost table



	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 10.33

Covering line 1

Covering line 2

Covering line 3

- This requires three lines to cover the zeros so the solution is optimal

Making the Final Assignment

- The optimal assignment is Adams to project 3, Brown to project 2, and Cooper to project 1
- But this is a simple problem
- For larger problems one approach to making the final assignment is to select a row or column that contains only one zero
- Make the assignment to that cell and rule out its row and column
- Follow this same approach for all the remaining cells

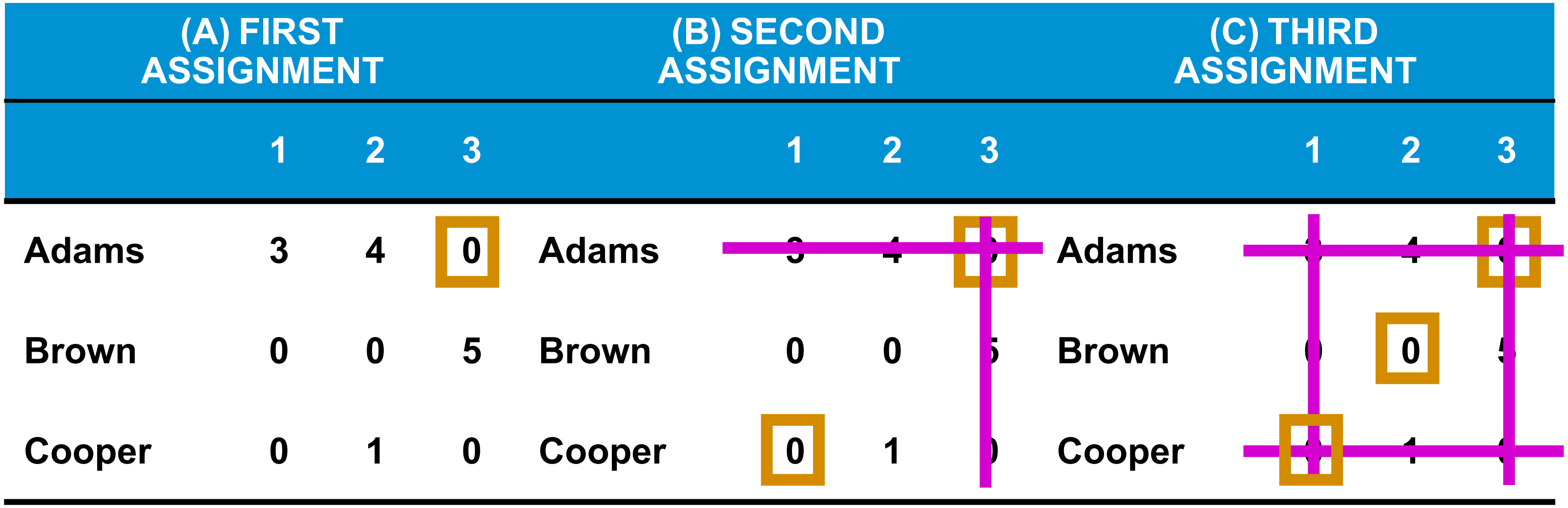
Making the Final Assignment

- Total labor costs of this assignment are

ASSIGNMENT	COST ($)
Adams to project 3	6
Brown to project 2	10
Cooper to project 1	9
Total cost	25

Making the Final Assignment

- **Making the final assignments**



(A) FIRST ASSIGNMENT				(B) SECOND ASSIGNMENT				(C) THIRD ASSIGNMENT			
	1	2	3		1	2	3		1	2	3
Adams	3	4	$\boxed{0}$	Adams	3	4	$\boxed{0}$	Adams	3	4	$\boxed{0}$
Brown	0	0	5	Brown	0	0	5	Brown	0	$\boxed{0}$	5
Cooper	0	1	0	Cooper	$\boxed{0}$	1	0	Cooper	$\boxed{0}$	1	0

Table 10.34

Unbalanced Assignment Problems

- Often the number of people or objects to be assigned does not equal the number of tasks or clients or machines listed in the columns, and the problem is *unbalanced*
- When this occurs, and there are more rows than columns, simply add a *dummy column* or task
- If the number of tasks exceeds the number of people available, we add a *dummy row*
- Since the dummy task or person is nonexistent, we enter zeros in its row or column as the cost or time estimate

Unbalanced Assignment Problems

- The Fix-It Shop has another worker available
- The shop owner still has the same basic problem of assigning workers to projects
- But the problem now needs a dummy column to balance the four workers and three projects

	PROJECT			
PERSON	1	2	3	DUMMY
Adams	$11	$14	$6	$0
Brown	8	10	11	0
Cooper	9	12	7	0
Davis	10	13	8	0

Table 10.35